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Поступила в редакцию 01.06.2026
Поступила после доработки 07.09.2026
Принята к публикации 10.09.2026

https://doi.org/10.53360/2788-7995-2026-3(23)-59

IRSTI: 65.13.13

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MATHEMATICAL MODELING OF THE FINE GRINDING PROCESS OF MEAT-AND-BONE RAW MATERIALS

Abstract: *Fine grinding sets the dispersion of the raw material and, through it, the structure and water-binding capacity of the minced mass, yet in conventional lines it is separated from preliminary grinding, which increases the number of machines and the specific energy demand. The article presents a mathematical model of the fine grinding of meat-and-bone raw materials in a combined unit where both stages run sequentially in one working space from a single drive. The model is built on the Bingham rheological body supplemented by an element describing the instantaneous irreversible deformation of bone inclusions, and is arranged as two series-connected contours. Summation of the contour shear rates gives a compact equation with equivalent (reduced) characteristics, coupled with the kinematics of the cutting mechanism, so that the power of the process is expressed through yield shear stress, cutting area, blade radius and rotational speed. Adequacy was verified against yield shear stress measured by penetration in eight runs with three replications: the calculated Fi = 0.29164 is lower than the tabulated Ftab = 1.02 at f1 = 3, f2 = 4, α = 0.1. At 16.7 s^{-1} the combined unit reached 151.3 kg/h against 96.3 kg/h, 68 % of particles fell within 100–150 μm against 36 %, and specific energy consumption decreased from 31.15 to 22.14 kW·h/t.*

Key words: *meat-and-bone raw materials; fine grinding; combined grinding unit; rheological model; Bingham body; yield shear stress; cutting working bodies; Fisher's criterion.*

Introduction

Grinding is the operation that sets the degree of dispersion of raw material and, through it, the structure of minced meat, its water-binding capacity, rheological behaviour and the energy demand of every subsequent stage of sausage production [1]. In the production schemes currently

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used, coarse and medium grinding are carried out in a mincer, whereas fine grinding is transferred to a separate machine – a cutter, an emulsifier or a colloid mill. This sequence is technologically well mastered, but it multiplies the number of machines in the line, lengthens the cycle and raises both the specific energy consumption and the thermal load on the product.

A review of the relation between the grinding process and the quality of ground meat shows that particle size distribution, the geometry of the knife–plate pair and the rotational speed of the cutting bodies act on product quality jointly rather than independently, so that a change in any single design parameter cannot be assessed in isolation [2]. Studies of modified cutting elements confirm this conclusion: a change in the geometry of the plate holes or in the rotor–stator knife profile simultaneously alters throughput, particle size distribution and the water-binding capacity of the ground mass [3, 4]. For meat-and-bone raw materials the effect is even more pronounced, since the clearance between the rotating knives and their rotational speed together determine the yield shear stress and the water-binding capacity of the finely ground product [5].

A promising direction in food engineering is therefore the development of combined machines in which several operations are performed within one working space. For meat processing this makes it possible to convert lump raw material into a finely dispersed mass by the sequential action of the feeding and cutting working bodies, and the design of the cutting mechanism becomes the decisive factor of process efficiency [6]. Interest in such solutions is reinforced by the need for a deeper use of meat co-products, including meat-and-bone raw materials – a valuable but technologically difficult source of protein and minerals [7, 8].

The design of such a unit, however, cannot be reduced to a constructive solution alone: it requires a mathematical description of the behaviour of the raw material under mechanical loading in the zone of cutting, deformation and shear. Meat-and-bone raw materials are heterogeneous systems containing muscle and connective tissue, fat, water, mineral components and bone inclusions, and under mechanical treatment they exhibit elastic, viscous and plastic properties at the same time [9]. Simple empirical dependencies are insufficient here; a rheological model is required that describes deformation both at the preliminary and at the fine grinding stage [10, 11].

Existing approaches to the modelling of the cutting of biological materials are focused mainly on a single cutting act and on the mechanics of blade–material contact [12]; models that describe the sequential passage of the product through two cutting zones of one unit, with a change of the rheological state of the material between them, are practically absent. This determined the direction of the present study.

The purpose of the study is to develop and verify a mathematical model of the fine grinding of meat-and-bone raw materials in a combined grinding unit that predicts yield shear stress and provides a calculation basis for the selection of rational operating parameters of the cutting working bodies.

The tasks were: to justify the choice of a rheological model for meat-and-bone raw material; to describe preliminary and fine grinding by a two-contour mechanical scheme; to derive calculation dependencies for shear rate and for the power of the process; and to assess the adequacy of the model by Fisher’s criterion.

Conditions and methods of the study

The object of the study was the combined grinding process of meat and meat-and-bone raw materials in a unit that joins the stages of preliminary and fine grinding. The subject of the study was the structural and mechanical properties of the material formed during its interaction with the cutting working bodies. Yield shear stress was chosen as the main evaluation parameter, since it characterises the resistance of the minced mass to deformation before flow begins and is directly related to the force on the blade.

The experimental unit includes a screw feeder and a cutting mechanism formed by two successive knife pairs mounted on a common shaft, each pair containing a two-blade knife. In one pass the raw material is therefore treated twice by a knife pair, and the cutting multiplicity is $i = 2 \cdot 2 = 4$. This double cycle, implemented in one machine and from one drive, is what makes the combination of preliminary and fine grinding possible. Beef of the first category at +2…+3 °C was used as raw material; the control scheme was a mincer followed by a separate fine grinding machine.

The modelling procedure assumes that the first stage corresponds to the preliminary destruction of lump material and the second to the fine dispersion of the already disintegrated mass: at the first stage the material retains pronounced viscoelastic properties, at the second it becomes

more plastic and flowable. The Bingham rheological body was therefore used as the basis and supplemented with an element describing the loss of strength of bone inclusions under cutting and shear.

Yield shear stress was determined by the penetration method on a conical plastometer [13]. The cone was immersed into the sample, the immersion depth was read after 180 s, and the yield shear stress of each sample was calculated as

$$\theta_0 = K\, m\, g\, /\, h^2, \quad (1)$$

where K is the constant of the cone (K = 2.1); m is the mass of the cone with all movable parts, kg; g is the acceleration of gravity, m/s^2; h is the depth of cone immersion, m.

Eight experimental runs were performed with three replications in each run; the rotational speed of the knives, the clearance between the knives and the configuration of the cutting elements were varied as factors. The arithmetic mean of the three measurements was taken as the experimental value of the run. Calculated values obtained from the model were compared with the experimental ones, and the adequacy of the model was assessed through the residual variance and the reproducibility variance using Fisher's criterion.

Notation: θ is shear stress, Pa; θ_T is the yield limit under shear, Pa; θ_H is the limit of instantaneous irreversible deformation of bone inclusions, Pa; G is the shear modulus, Pa; η is plastic (shear) viscosity, Pa·s; γ is shear deformation; dγ/dτ is shear rate, s^{-1}; τ is time, s. Subscripts 1 and 2 refer to the preliminary and fine grinding stage.

Results of the study

Rheological scheme of the combined process. The combined grinding process is represented as a sequential deformation of meat-and-bone raw material in two connected zones – the preliminary grinding zone and the fine grinding zone – each characterised by its own elasticity, viscosity and yield limit. The corresponding mechanical scheme is shown in Figure 1.

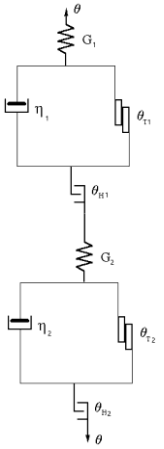


Figure 1 – Mechanical rheological scheme of the combined grinding process of meat-and-bone raw materials

The first contour describes preliminary grinding and includes the parameters G_1, η_1, θ_{T1} and θ_{H1}; the second contour describes fine grinding and includes G_2, η_2, θ_{T2} and θ_{H2}. Such a structure allows the resistance of the material at the stage of initial destruction and its behaviour during subsequent fine dispersion to be considered separately.

The total deformation of meat-and-bone material is treated as the sum of the deformation of the Bingham body and the deformation of the element that represents the behaviour of bone inclusions during grinding:

$$\gamma = \gamma_B + \gamma_H. \quad (2)$$

After differentiation with respect to time the total shear rate is written as the sum of two components:

$$d\gamma/d\tau = d\gamma_B/d\tau + d\gamma_H/d\tau. \quad (3)$$

For the Bingham element the deformation rate is determined by the elastic and the viscoplastic components. When the yield limit is reached the material begins to deform irreversibly, and the shear rate depends on the difference between the applied stress and the threshold stresses.

If $\theta \le \theta_H$, the body follows the Bingham model; if $\theta \ge \theta_H$, the component of instantaneous irreversible deformation of bone inclusions is included in the calculation. For the first contour the shear rate is expressed as

$$d\gamma_1/d\tau = (\theta - \theta_{T1} - \theta_{H1})/\eta_1 + (d\theta/d\tau)/G_1, \quad (4)$$

and for the second contour, analogously,

$$d\gamma_2/d\tau = (\theta - \theta_{T2} - \theta_{H2})/\eta_2 + (d\theta/d\tau)/G_2. \quad (5)$$

Since preliminary and fine grinding are performed consecutively within one unit, the total shear rate is determined by the sum of the contributions of both contours:

$$d\gamma/d\tau = d\gamma_1/d\tau + d\gamma_2/d\tau, \quad (6)$$

which, with account of expressions (4) and (5), gives

$$d\gamma/d\tau = (\theta - \theta_{T1} - \theta_{H1})/\eta_1 + (\theta - \theta_{T2} - \theta_{H2})/\eta_2 + (d\theta/d\tau)(1/G_1 + 1/G_2). \quad (7)$$

Introducing the equivalent (reduced) characteristics of the two-contour model

$$1/\eta_e = 1/\eta_1 + 1/\eta_2; \quad 1/G_e = 1/G_1 + 1/G_2, \quad (8)$$

and the equivalent limit of deformation

$$\theta_e = \eta_e [(\theta_{T1} + \theta_{H1})/\eta_1 + (\theta_{T2} + \theta_{H2})/\eta_2], \quad (9)$$

the rheological equation of the combined grinding process takes the compact form

$$d\gamma/d\tau = (\theta - \theta_e)/\eta_e + (d\theta/d\tau)/G_e. \quad (10)$$

Equation (10) is formally identical with the equation of a single Bingham body, but its coefficients are determined by the parameters of both stages: unlike a single-stage description, the change of the rheological state of the material between the two cutting zones is retained in the equivalent characteristics and is not averaged out.

Coupling with the kinematics of the cutting mechanism. To pass from the rheological description to design parameters, the model was linked to the kinematics of the cutting mechanism. The cutting speed is determined by the angular velocity and the radius of the cutting body,

$$v = \omega R = 2\pi n R, \quad (11)$$

where n is the rotational frequency of the knives, s^{-1}, and R is the radius of the cutting body, m. The force required to destroy the raw material in the cutting zone is $F = \theta_0 S$, the work of cutting is $A = Fl$, and the power of the process, $N = A/t$, after substitution takes the form

$$N = \theta_0 S v = 2\pi \theta_0 S n R, \quad (12)$$

where S is the area of the cut surface, m^2. Expression (12) directly connects the rheological characteristic of the product, obtained from equation (10), with the design and operating parameters of the cutting mechanism, which makes the model usable for engineering calculation.

Verification of the model. Model verification was based on experimental values of yield shear stress. Eight experimental runs were performed with three measurements in each; the results are presented in Table 1.

Table 1 – Experimental values of yield shear stress, Pa

Run	θ_1	θ_2	θ_3	θavg
1	706.12	705.04	703.10	704.75
2	741.40	740.54	745.32	742.42
3	669.50	664.90	665.76	666.72
4	638.90	636.54	632.18	635.87
5	743.70	742.42	745.34	743.82
6	782.21	783.38	778.10	781.23
7	709.68	710.60	713.50	711.26
8	679.15	677.30	674.30	676.92

The experimental values ranged from 635.87 Pa (run 4) to 781.23 Pa (run 6), which confirms that the structural and mechanical properties of the minced mass are sensitive to the conditions of mechanical treatment. The spread between repeated measurements within one run did not exceed 1.1 % of the mean, which indicates satisfactory reproducibility.

The calculated values of yield shear stress obtained from the model are given in Table 2.

Table 2 – Calculated values of yield shear stress, Pa

Run	1	2	3	4	5	6	7	8
θcalc	704.755	742.439	666.739	635.905	743.824	781.247	711.290	676.982

Comparison of calculated and experimental values shows close agreement for all runs: the deviation does not exceed 0.01 % and does not change the general trend of yield shear stress variation. The residual variance was determined as

$$S_0^2 = m \, \Sigma \, (\theta_{calc,i} - \theta_{avg,i})^2 / (Z_k - 1), \quad (13)$$

and the reproducibility variance as

$$S_{rep}^2 = \Sigma \, S_i^2 / (Z_k - 1). \quad (14)$$

where m is the number of parallel measurements; Z_k is the number of runs entering the corresponding estimate; S_i^2 is the sample variance of the i-th run. The degrees of freedom of the residual and the reproducibility variance were $f_1 = 3$ and $f_2 = 4$.

The obtained values were $S_0^2 = 275.058$ and $S_{rep}^2 = 943.141$. Fisher's criterion was calculated as the ratio

$$Fi = S_0^2 / S_{rep}^2. \quad (15)$$

The calculated value $Fi = 0.29164$ is lower than the tabulated value $F_{tab} = 1.02$ at $f_1 = 3$, $f_2 = 4$ and $\alpha = 0.1$, hence the proposed model is adequate to the experimental data.

Discussion of scientific results

The practical relevance of the model is confirmed by the technological indicators of the combined unit obtained at the rational rotational speed of the cutting bodies n = 16.7 s^{-1} (1000 min^{-1}). The comparison with the control scheme, in which the same volume of raw material was processed first in a mincer and then in a separate fine grinding machine, is given in Table 3.

Table 3 – Comparison of the combined unit with the control grinding scheme

Indicator	Control scheme	Combined unit	Change
Productivity, kg/h	96.3	151.3	+57.1 %
Power consumed, kW	3.00	3.35	+11.7 %
Specific energy consumption, kW·h/t	31.15	22.14	−28.9 %
Specific productivity, kg/(h·kW)	32.10	45.16	+40.7 %
Share of 100-150 μm particles, %	36	68	+32 p.p.
Raw material temperature before grinding, °C	+2…+3	+2…+3	-

The data of Table 3 show that the combination of the two grinding stages in one working process improves the dispersion quality and increases the productivity of the equipment without a proportional growth of the installed power: the power consumed rises by 11.7%, whereas the productivity rises by 57.1%, so that the specific energy consumption decreases by 28.9%.

The calculation of particle size from the cutting multiplicity of the mechanism ($i = 4$) gives 141 μm, which lies inside the experimentally established range of 100-150 μm where 68% of the particles of the finely ground mass are concentrated. Together with the agreement of the calculated and experimental yield shear stress, this indicates that expressions (10) and (12) may be used for the selection of design parameters of the cutting mechanism and not only for the interpretation of the experiment.

The results are consistent with the data of other authors. It has been shown earlier that for finely ground chicken bone the yield shear stress and the water-binding capacity depend jointly on the speed of the knives and the clearance between them [5]; the two-contour model explains this, since both factors enter the equivalent characteristics (8) and (9). The improvement of the granulometric composition obtained here is of the same order as that reported for modified plates with variable hole diameters and for modified rotor–stator knives of a colloid mill [3, 4, 14], which indicates that the effect is produced by the geometry and the multiplicity of cutting rather than by an increase of the rotational speed alone. This is important in practice, because an increase of the rotational speed is the simplest but the least favourable way of intensification: it raises the energy consumption and the temperature of the product, and above a certain speed the heating of the mass leads to protein denaturation [4].

From the engineering point of view it is essential that the model accounts not only for the viscoplastic behaviour of the meat raw material but also for bone inclusions, which lose strength under cutting and shear. The threshold θ_H introduced into each contour is what distinguishes meat-and-bone raw material from a homogeneous minced system, and its omission underestimates the resistance of the material in the first cutting zone.

The model does not replace the experimental adjustment of equipment, but it reduces the volume of preliminary trials and supports the transition from an empirical selection of processing modes to a calculation-based justification. Its main limitation is that the raw material temperature was kept at +2…+3 °C and the temperature dependence of the rheological parameters is not included in equations (8)-(10), although the thermal state of minced meat is known to change its behaviour considerably [15]. Further development may therefore introduce temperature, knife configuration and inter-knife clearance as explicit arguments of the equivalent characteristics.

Conclusion

A mathematical model of the fine grinding of meat-and-bone raw materials has been developed on the basis of the Bingham rheological body adapted to combined preliminary and fine grinding; it accounts for the elastic, viscous and plastic properties of the material and for the limit of instantaneous irreversible deformation of bone inclusions.

The process is represented as two series-connected contours describing, respectively, the preliminary grinding of lump raw material and the fine grinding of the pre-disintegrated mass. Summation of the contour shear rates and the introduction of equivalent (reduced) characteristics give the compact rheological equation (10), which retains the change of state of the material between the two cutting zones; its coupling with the kinematics of the cutting mechanism yields the calculation dependence (12) for the power of the process.

The adequacy of the model was confirmed by comparing calculated and experimental values of yield shear stress: the calculated Fisher criterion Fi = 0.29164 is lower than the tabulated value Ftab = 1.02 at f1 = 3, f2 = 4 and α = 0.1, and the deviation of the calculated values from the experimental ones does not exceed 0.01 %.

At the rational rotational speed of 16.7 s^{-1} the combined unit provided 151.3 kg/h against 96.3 kg/h for the control scheme, 68 % of particles within the 100–150 μm range against 36 %, and a decrease of the specific energy consumption from 31.15 to 22.14 kW·h/t. The model may therefore be used in the design and optimisation of the cutting working bodies of combined grinding units.

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ПРОЦЕССА ТОНКОГО ИЗМЕЛЬЧЕНИЯ МЯСОКОСТНОГО СЫРЬЯ

В статье представлена математическая модель процесса тонкого измельчения мясокостного сырья в комбинированной установке, в которой стадии мелкого и тонкого измельчения выполняются последовательно в одном рабочем пространстве и от одного привода. Модель построена на основе реологического тела Бингама, дополненного элементом, описывающим мгновенно-необратимую деформацию костных включений, и представлена в виде двух последовательно соединённых контуров, каждый из которых характеризуется собственными модулем упругости при сдвиге, пластической вязкостью, пределом текучести и пределом мгновенно-необратимой деформации. Суммирование скоростей сдвига по контурам приводит к компактному уравнению с эквивалентными (приведёнными) характеристиками, которое связывается с кинематикой режущего механизма, что позволяет выразить мощность процесса через предельное напряжение сдвига, площадь резания, радиус и частоту вращения ножей. Адекватность проверена по значениям предельного напряжения сдвига, определённым методом пенетрации в восьми опытах с трёхкратной повторностью: расчётный критерий Фишера Fi = 0,29164 меньше табличного Fтаб = 1,02 при f1 = 3, f2 = 4, α = 0,1. При рациональной частоте вращения ножей 16,7 с⁻¹ производительность установки составила 151,3 кг/ч против 96,3 кг/ч, доля частиц 100-150 мкм – 68% против 36%, удельные энергозатраты снизились с 31,15 до 22,14 кВт·ч/т.

Ключевые слова: *мясокостное сырьё; тонкое измельчение; комбинированная установка; реологическая модель; тело Бингама; предельное напряжение сдвига; режущие рабочие органы; критерий Фишера.*

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ЕТ-СҮЙЕКТІ ШИКІЗАТТЫ ЖҰҚА МАЙДАЛАУ ПРОЦЕСІН МАТЕМАТИКАЛЫҚ МОДЕЛЬДЕУ

Мақалада ұсақ және жұқа майдалау сатылары бір жұмыс кеңістігінде әрі бір жетектен дәйекті түрде орындалатын құрама қондырғыда ет-сүйекті шикізатты жұқа майдалау процесінің математикалық моделі ұсынылған. Модель Бингамның реологиялық денесі негізінде құрылған, сүйек қосындыларының лездік қайтымсыз деформациясын сипаттайтын элементпен толықтырылған және дәйекті жалғанған екі контур түрінде берілген; әрбір контур ығысу кезіндегі серпімділік модулімен, пластикалық тұтқырлығымен, аққыштық шегімен және лездік қайтымсыз деформация шегімен сипатталады. Контурлар бойынша ығысу жылдамдықтарын қосу эквивалентті (келтірілген) сипаттамалары бар ықшам теңдеуге әкеледі; бұл теңдеу кескіш механизмнің кинематикасымен байланыстырылып, процесс қуатын шекті ығысу кернеуі, кесу ауданы, пышақтардың радиусы мен айналу жиілігі арқылы өрнектеуге мүмкіндік береді. Модельдің сәйкестігі пенетрация әдісімен үш мәрте қайталанған сегіз тәжірибеде тексерілді: есептік Фишер критерийі Fi = 0,29164 кестелік Fкест = 1,02 мәнінен төмен (f1 = 3, f2 = 4, α = 0,1). Пышақтардың 16,7 $с^{-1}$ ұтымды айналу жиілігінде қондырғының өнімділігі 151,3 кг/сағ болды (бақылауда – 96,3 кг/сағ), 100-150 мкм бөлшектердің үлесі 36 %-ға қарсы 68 % құрады, меншікті энергия шығыны 31,15-тен 22,14 кВт·сағ/т-ға дейін төмендеді.

Түйін сөздер: *ет-сүйекті шикізат; жұқа майдалау; құрама қондырғы; реологиялық модель; Бингам денесі; шекті ығысу кернеуі; кескіш жұмыс органдары; Фишер критерийі.*

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Received 20.06.2026
Revised 04.09.2026
Accepted 09.09.2026