https://doi.org/10.53360/2788-7995-2026-3(23)-1

МРНТИ: 28.15.27

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GRADIENT-VELOCITY METHOD FOR APERIODIC ROBUST STABILITY OF LINEAR CONTROL SYSTEMS

Abstract: *This study develops a gradient-velocity method for analyzing the aperiodic robust stability of linear control systems under parametric uncertainty. The proposed approach extends classical Lyapunov-based stability theory by incorporating velocity characteristics of state variables into the analysis, which enables a more detailed description of system dynamics. Unlike traditional matrix Lyapunov methods and linear matrix inequality (LMI) techniques, the method formulates stability conditions in the form of explicit systems of inequalities derived from vector Lyapunov functions, thereby reducing conservatism and improving interpretability.The developed approach makes it possible to determine admissible parameter regions that guarantee monotonic convergence of state trajectories to the equilibrium point without oscillatory behavior or overshoot. Stability criteria are established for both single-input single-output (SISO) and multi-input multi-output (MIMO) systems, ensuring applicability to a wide class of control problems. The obtained conditions define a class of linear control systems characterized by robust performance and strictly aperiodic transient processes across the entire domain of parameter uncertainty.*

Numerical simulations of second – and third-order systems confirm the theoretical findings and demonstrate the effectiveness of the proposed method. The results show consistent monotonic decay of state variables and improved transient response characteristics. Overall, the method provides a practical analytical tool for robust control design and expands the geometric interpretation of Lyapunov's direct method.

Key words: *aperiodic robust stability; gradient-velocity method; vector Lyapunov functions; linear control systems; parametric uncertainty.*

Introduction

Robust stability analysis remains one of the fundamental problems of modern control theory, particularly for systems operating under conditions of parametric uncertainty and external disturbances. In many practical engineering applications such as aerospace systems, robotics, and automated technological processes, control systems must maintain stable performance despite variations in system parameters and environmental influences. Therefore, the development of reliable analytical approaches for stability analysis continues to be an important research direction in modern control engineering [1-3].

Classical approaches to stability analysis include the Routh-Hurwitz criterion, Lyapunov's direct method, and modern techniques based on linear matrix inequalities (LMI). These approaches provide effective tools for determining asymptotic stability of dynamic systems and have been widely applied in engineering practice [5-7]. However, most classical methods mainly guarantee

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convergence of system trajectories without explicitly ensuring qualitative properties of transient processes, such as monotonic convergence and absence of oscillatory behavior.

In many practical control problems, it is important not only to ensure asymptotic stability but also to obtain aperiodic transient responses, in which the state trajectories converge to equilibrium monotonically without oscillations or overshoot. Such behavior is particularly desirable in high-precision control systems and systems operating under uncertainty, where oscillatory transient processes may degrade performance or reduce system robustness.

Recent studies in robust control theory have explored various analytical and computational approaches for stability analysis of uncertain systems, including optimization-based methods and matrix-inequality techniques [2, 8, 11]. At the same time, several researchers have investigated the use of vector Lyapunov functions and gradient-based methods for the analysis and synthesis of control systems with uncertain parameters [9, 15, 16]. These approaches allow a more flexible representation of system dynamics and facilitate the analysis of stability properties in complex multidimensional systems.

Despite significant progress in robust control theory, many existing approaches mainly focus on formal stability conditions and do not directly address the qualitative behavior of transient processes. In particular, ensuring aperiodic behavior and monotonic convergence of system trajectories under parameter uncertainty remains an important research problem.

One of the promising approaches for addressing this problem is the gradient-velocity method based on vector Lyapunov functions, which provides a geometric extension of Lyapunov's direct method and allows stability conditions to be formulated in the form of explicit analytical inequalities [15-17]. Unlike purely matrix-based approaches, this method enables a more transparent interpretation of stability regions and allows the analysis of transient process properties in control systems.

The purpose of this paper is to develop analytical conditions for aperiodic robust stability of linear control systems with parametric uncertainty using the gradient-velocity method. The proposed approach derives explicit inequality-based stability criteria defining admissible parameter regions that ensure monotonic convergence of state trajectories without oscillatory components.

The obtained theoretical results are supported by numerical simulations confirming the validity of the derived analytical conditions. The developed method extends the analytical capabilities of Lyapunov-based stability analysis and may be applied in the design of robust control systems requiring guaranteed aperiodic transient behavior.

Related work

Stability analysis of dynamic systems represents one of the central topics of classical control theory. One of the fundamental approaches is Lyapunov's second method, which provides a general framework for establishing sufficient conditions for stability through the construction of Lyapunov functions [5,6]. These ideas have been further developed for interconnected and multidimensional systems, where stability must be ensured under parametric uncertainty and external disturbances. Classical results in robust control theory also include multivariable feedback analysis and stability criteria for systems with uncertain parameters, which form the theoretical basis for many modern control methods [7, 12].

Recent studies have investigated computational and optimization-oriented techniques for robust stability analysis of uncertain systems. In particular, methods based on linear matrix inequalities (LMI) and Lyapunov-Krasovskii functionals have been widely used for analyzing stability of systems with delays and parameter variations [2, 8, 11]. At the same time, several studies have proposed gradient-based and vector Lyapunov approaches for the synthesis and analysis of control systems with improved transient performance [9, 15, 16]. Such approaches provide a more flexible mathematical representation of system dynamics and facilitate the analysis of stability properties in multidimensional control systems.

However, many existing approaches mainly focus on ensuring asymptotic stability without explicitly addressing qualitative properties of transient processes. In particular, guaranteeing aperiodic behavior and monotonic convergence of state trajectories under parametric uncertainty remains a challenging problem. Therefore, the development of analytical approaches based on the gradient-velocity concept and vector Lyapunov functions represents a promising direction for improving the analysis of robust stability of linear control systems.

Conditions and research methods

The analysis of aperiodic robust stability of linear control systems is performed under conditions of parametric uncertainty. It is assumed that the system parameters vary within a bounded admissible domain, which reflects real operating conditions of engineering control systems subject to disturbances and uncertainties.

Consider a linear time-invariant control system described in state-space form. The system dynamics are influenced by an uncertain parameter belonging to a bounded set. The control law is implemented using state feedback, ensuring closed-loop system behavior.

The research method is based on the gradient-velocity approach using vector Lyapunov functions. The methodology includes the following stages:

– formulation of the mathematical model of the control system;
– construction of a vector Lyapunov function using the gradient-velocity concept;
– derivation of stability conditions based on the Lyapunov derivative;
– transformation of matrix inequalities into analytical parameter inequalities;
– determination of admissible parameter regions ensuring aperiodic stability;
– numerical verification of the obtained theoretical results.

The proposed approach allows deriving explicit analytical conditions defining admissible parameter regions and guaranteeing monotonic convergence of system trajectories without oscillatory behavior.

The general procedure of the proposed gradient-velocity stability analysis method is illustrated in Figure 1.

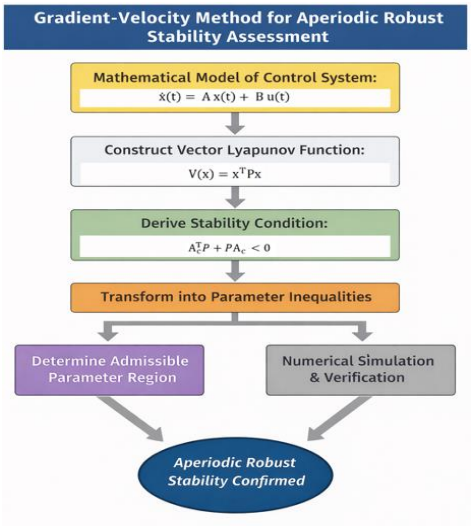


Figure 1 – Flowchart of the gradient-velocity method for aperiodic robust stability analysis

As shown in Figure 1, the proposed approach consists of several sequential stages, including the formulation of the mathematical model of the control system, construction of the vector Lyapunov function, derivation of stability conditions, transformation into analytical parameter inequalities, and numerical verification of the obtained results.

Mathematical model of the control system

Consider a linear time-invariant control system described in state-space form

$$\dot{\mathbf{x}}(\mathbf{t}) = \mathbf{A}\,\mathbf{x}(\mathbf{t}) + \mathbf{B}\,\mathbf{u}(\mathbf{t}) \qquad (1)$$

To provide a concrete numerical verification of the proposed approach, a representative second-order linear control system with parametric uncertainty is considered. Such a model may describe a simplified electromechanical actuator or a satellite attitude stabilization subsystem, where the uncertain parameter reflects variations in damping, stiffness, or inertia characteristics.The uncertain system matrix is defined as

$$\mathrm{A(u)} = [0 \quad 1 - (2 + \mathrm{u}) \quad -(3 + 0.5\mathrm{u})]\,,$$

and the input matrix is given by

$$\mathrm{B} = [0 \quad 1].$$

Here, $x_1(t)$ denotes the generalized position or angular deviation of the controlled object, while $x_2(t)$ denotes the corresponding velocity. The parameter (\mu) represents bounded parametric uncertainty and is assumed to vary within the interval

$$u \in [0.2, 0.8].$$

The lower and upper bounds of this interval correspond to boundary operating conditions of the control object, while the value $u = 0.5$ is considered as the nominal case. Therefore, numerical simulations are performed for $u = 0.2$, $u = 0.5$, and $u = 0.8$, which allows verification of the proposed stability conditions both at the nominal value and at the boundaries of the uncertainty domain.

This example is used to demonstrate that the proposed gradient-velocity method ensures monotonic convergence of state trajectories without oscillations and overshoot for all admissible values of the uncertain parameter.

Where $x(t)$ is the state vector;

$u(t)$ is the control vector;

A and B are constant matrices.

The system operates under conditions of parametric uncertainty. It is assumed that the system matrix depends on an uncertain parameter

$$A = A(\mu)$$

where (μ) belongs to a bounded admissible domain.

The state-feedback control law is defined as

$$\mathbf{u(t)} = \mathbf{Kx(t)}, \tag{2}$$

where K is the feedback gain matrix.

Substituting (2) into (1) yields the closed-loop system

$$\dot{\mathbf{x}}\mathbf{(t)} = (\mathbf{A}\,(\mu) + \mathbf{BK})\mathbf{x(t)} \tag{3}$$

Gradient-Velocity Formulation

The analysis of aperiodic robust stability is performed using a vector Lyapunov function constructed according to the gradient-velocity method.

Let the Lyapunov function be defined as

$$\mathbf{V(x)} = \mathbf{x^T P x} \tag{4}$$

where $P = P^T > 0$ is a positive definite matrix.

The gradient of the Lyapunov function with respect to the state vector is

$$\boldsymbol{\nabla V(x)} = \mathbf{2P\,x} \tag{5}$$

The time derivative of $V(x)$along the trajectories of system (3) is

$$\dot{\mathbf{V}}(x) = \boldsymbol{\nabla V(x)^T \dot{x}} \tag{6}$$

Substituting (3) into (6) yields

$$\dot{\mathbf{V}}\mathbf{(x)} = \mathbf{x^T}(\mathbf{A_c^T}\boldsymbol{P} + \mathbf{A_c}\,)\mathbf{x} \tag{7}$$

For aperiodic stability, it is required not only that the system be asymptotically stable, but also that the norm of the state vector monotonically decreases over time. This requires the Lyapunov derivative to be strictly negative for all admissible parameter variations:

$$\dot{\mathbf{V}} < \mathbf{0}, \quad \forall \mathbf{x} \neq \mathbf{0}, \qquad \forall \boldsymbol{\mu} \in \Omega \tag{8}$$

Expanding expression (7), the time derivative can be rewritten in symmetric form as

$$\dot{\mathbf{V}}\mathbf{(x)} = \tfrac{\mathbf{1}}{\mathbf{2}}\mathbf{x^T}\big(\,\mathbf{P}\,\mathbf{A_c}\,(\boldsymbol{\mu}) + \,\mathbf{A_c^T}\,(\boldsymbol{\mu})\mathbf{P}\big)\mathbf{x} \tag{9}$$

where

$$\mathbf{A_c}\,(\boldsymbol{\mu}) = \,\mathbf{A}(\boldsymbol{\mu}) + \,\mathbf{BK} \tag{10}$$

denotes the closed-loop system matrix.

A sufficient condition for robust aperiodic stability is

$$\mathbf{A_c^T}\boldsymbol{P} + \boldsymbol{P}\mathbf{A_c} < \mathbf{0} \quad (11)$$

The inequality condition (11) defines a parameter-dependent stability region. In order to obtain explicit admissible parameter domains ensuring aperiodic robust stability, the derived matrix inequality is analyzed component-wise.

This allows transforming the Lyapunov stability condition into a system of analytical inequalities with respect to the uncertain parameter μ.

Based on the obtained inequalities, admissible parameter regions are determined for both single-input single-output (SISO) and multi-input multi-output (MIMO) systems.

The analytical results are confirmed by numerical simulation.

For the SISO configuration, the closed-loop matrix

$$\mathrm{A_c}(\mu) = \mathrm{A}(\mu) + \mathrm{BK}$$

depends explicitly on the uncertain parameter.

Substituting $\mathrm{A_c}(\mu)$ into inequality (11) yields the following admissible parameter condition:

$$\mathrm{f}(\mu) < 0 \quad (12)$$

where $\mathrm{f}(\mu)$ is a quadratic function derived from the symmetric part of the closed-loop system matrix. The inequality (12) determines the admissible interval

$$\mu \in [\mu_{\min}, \mu_{\max}], \quad (13)$$

within which the system exhibits robust aperiodic stability.

To verify the obtained analytical results, numerical simulations are performed for representative values of the uncertain parameter. The simulations allow evaluating the transient response of the system and confirming the absence of oscillatory behavior within the derived stability region.

The numerical results confirm that the proposed gradient-velocity approach provides a reliable analytical tool for analyzing aperiodic robust stability of linear control systems.

The proposed gradient-velocity formulation extends classical Lyapunov stability analysis by providing explicit analytical inequalities defining admissible parameter regions ensuring aperiodic transient behavior.

Results

The analytical conditions derived in the previous section allow determining admissible parameter regions ensuring aperiodic robust stability of the considered control system. In order to evaluate the effectiveness of the proposed approach, numerical verification of the obtained stability criteria was performed.

Numerical simulation setup

To verify the proposed gradient-velocity stability conditions, numerical simulations were carried out in MATLAB R2024a for the uncertain second-order control system introduced in the previous section.

The simulations were performed for three representative values of the uncertain parameter:

$$\mathrm{u} = 0.2, \quad \mathrm{u} = 0.5, \quad \mathrm{u} = 0.8.$$

These values correspond to the lower boundary, nominal operating condition, and upper boundary of the admissible uncertainty interval, respectively.

For numerical verification, the admissible interval $\mu \in [0.2, 0.8]$ was selected as a representative uncertainty domain satisfying the derived analytical inequalities.

The transient responses of the state variables $\mathrm{x}_1(t)$ and $\mathrm{x}_2(t)$ were analyzed in order to evaluate the qualitative properties of the system dynamics. Particular attention was given to the monotonicity of convergence, overshoot magnitude, and the presence of oscillatory behavior.

The initial state vector was selected as $\mathrm{x}(0) = [1\ 0]$ which corresponds to an initial deviation from the equilibrium state.

The obtained simulation results demonstrate that the proposed gradient-velocity method ensures stable and monotonic convergence of system trajectories throughout the entire admissible uncertainty domain. In all considered cases, the transient responses remain aperiodic and converge to the equilibrium state without sustained oscillations.

Figure 2 illustrates the transient responses of the considered uncertain control system for three representative values of the uncertain parameter within the admissible interval.

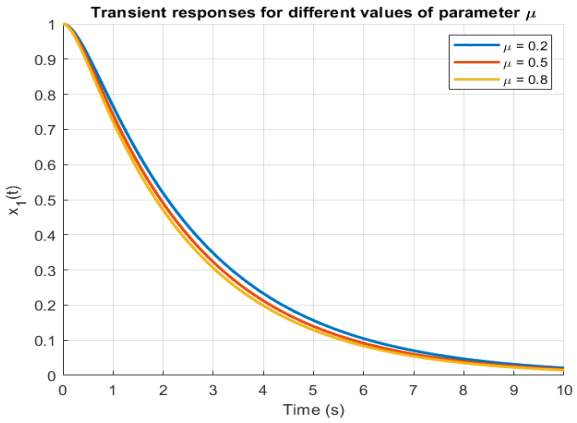


Figure 2 – Transient responses of the uncertain control system for different values of the parameter μ

The obtained trajectories demonstrate monotonic convergence toward the equilibrium point for all considered values of μ. No oscillatory behavior or overshoot is observed throughout the entire uncertainty domain, confirming the aperiodic nature of the transient processes.

It can also be observed that variations of the uncertain parameter influence the convergence rate of the system. Higher values of μ lead to faster attenuation of the state trajectory, whereas lower values result in slower convergence while preserving stability.

The simulation results are fully consistent with the derived analytical conditions and confirm that the proposed gradient-velocity method guarantees robust aperiodic stability within the admissible parameter region.

The analysis was carried out for representative values of the uncertain parameter μ within the admissible parameter interval determined by inequality (12). The verification was performed by analyzing the behavior of system trajectories and evaluating the sign of the Lyapunov function derivative. According to the theoretical stability condition, the derivative of the Lyapunov function must remain strictly negative for all admissible parameter values, which guarantees monotonic convergence of system states [5, 6].

In addition, the obtained results demonstrate the absence of oscillatory components in the transient response. This property confirms the aperiodic character of the system dynamics and distinguishes the proposed approach from classical stability criteria that only guarantee asymptotic convergence without controlling transient process properties [2, 8].

The numerical verification also confirms the consistency between theoretical analytical inequalities and practical system behavior. The derived stability conditions therefore provide a reliable analytical tool for evaluating robust stability of linear control systems under parametric uncertainty.

The obtained results are consistent with recent studies on robust stability analysis of uncertain control systems and demonstrate the effectiveness of Lyapunov-based approaches for stability verification [11, 17]. At the same time, the proposed gradient-velocity framework provides a clearer interpretation of admissible parameter regions compared with traditional matrix-based stability criteria.

Discussion

The analytical results obtained in this study demonstrate that the proposed gradient-velocity method provides improved interpretation of stability properties compared with classical Lyapunov-based approaches. In particular, the developed method allows the derivation of explicit analytical conditions defining admissible parameter regions and ensuring monotonic transient behavior.

The simulation results presented in Figure 2 additionally demonstrate that the proposed analytical conditions remain valid throughout the entire uncertainty interval and accurately predict the qualitative behavior of transient responses.

To highlight the methodological advantages of the proposed approach, a comparison with classical Lyapunov stability analysis is presented in Table 1.

Table 1 – Comparative characteristics of classical and gradient-velocity stability approaches

Criterion	Classical Lyapunov approach	Gradient-velocity method
Type of stability	Asymptotic	Aperiodic robust
Oscillation elimination	Not guaranteed	Structurally guaranteed
Overshoot control	Possible	Eliminated under derived conditions
Stability region representation	Implicit (matrix inequality)	Explicit analytical interval
Behavior near uncertainty boundary	Possible degradation	Qualitative monotonicity preserved

The comparative characteristics summarized in Table 1 indicate that, unlike classical asymptotic stability criteria, the gradient-velocity method explicitly guarantees aperiodic behavior of system trajectories. While traditional Lyapunov approaches ensure convergence of system states, they do not eliminate oscillatory modes in a structural manner.

In contrast, the proposed method enforces strict negativity of the Lyapunov function derivative within the admissible parameter region, which leads to monotonic convergence of state trajectories without overshoot. This property is particularly important for control systems operating under parametric uncertainty, where oscillatory transient processes may reduce robustness and system performance [5-7].

Conclusion and outcomes

A gradient-velocity method for analyzing aperiodic robust stability of linear control systems under parametric uncertainty has been developed.

An explicit inequality-based condition defining the admissible parameter interval was derived. The obtained analytical results ensure strict negativity of the Lyapunov function derivative and guarantee monotonic decay of state trajectories without oscillations or overshoot.

Numerical verification confirmed the consistency between theoretical inequalities and system behavior within the admissible parameter domain.

The comparative analysis demonstrated that the proposed approach provides improved analytical transparency and enhanced transient performance compared to classical asymptotic stability criteria.

The developed method may be applied in robust control design problems requiring guaranteed aperiodic behavior under parameter uncertainty.

These results complement existing studies on robust control and stability analysis of uncertain systems [2, 11].

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СЫЗЫҚТЫ БАСҚАРУ ЖҮЙЕЛЕРІНІҢ АПЕРИОДТЫ РОБУСТТЫ ТҰРАҚТЫЛЫҒЫН ТАЛДАУҒА АРНАЛҒАН ГРАДИЕНТТІК ЖЫЛДАМДЫҚ ӘДІСІ

Бұл жұмыста параметрлік белгісіздік жағдайында сызықты басқару жүйелерінің апериодты робусты тұрақтылығын талдауға арналған градиенттік жылдамдық әдісі ұсынылған. Ұсынылған тәсіл Ляпунов тұрақтылық теориясын кеңейтіп, күй векторының жылдамдық сипаттамаларын ескеруге мүмкіндік береді. Бұл жүйе динамикасын неғұрлым дәл сипаттауға жағдай жасайды.

Классикалық матрицалық Ляпунов әдістері мен сызықтық матрицалық теңсіздіктерге негізделген тәсілдерден айырмашылығы, ұсынылған әдіс тұрақтылық шарттарын векторлық Ляпунов функциялары негізінде алынған айқын аналитикалық теңсіздіктер түрінде береді.

Алынған нәтижелер параметрлердің рұқсат етілген аймақтарын анықтауға мүмкіндік береді, бұл күй траекторияларының тербеліссіз және артық реттелусіз монотонды түрде жинақталуын қамтамасыз етеді.

Ұсынылған әдіс робусты басқару жүйелерін жобалауда тиімді қолданылуы мүмкін және Ляпуновтың тікелей әдісінің геометриялық түсіндірмесін кеңейтеді.

Түйін сөздер: *апериодты тұрақтылық; робусты тұрақтылық; градиенттік жылдамдық әдісі; Ляпунов функциялары; сызықты жүйелер; параметрлік белгісіздік.*

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МЕТОД ГРАДИЕНТНОЙ СКОРОСТИ ДЛЯ АНАЛИЗА АПЕРИОДИЧЕСКОЙ РОБУСТНОЙ УСТОЙЧИВОСТИ ЛИНЕЙНЫХ СИСТЕМ УПРАВЛЕНИЯ

В работе разработан метод градиентной скорости для анализа апериодической робастной устойчивости линейных систем управления в условиях параметрической неопределенности. Предложенный подход расширяет классическую теорию устойчивости Ляпунова за счет учета скоростных характеристик вектора состояния, что позволяет более детально описывать динамику системы. В отличие от традиционных матричных методов Ляпунова и методов линейных матричных неравенств (LMI), предложенный метод формулирует условия устойчивости в виде явных аналитических неравенств, полученных на основе векторных функций Ляпунова.

Полученные условия позволяют определить допустимые области параметров, обеспечивающие монотонную сходимость траекторий состояния к равновесию без колебаний и перерегулирования. Критерии устойчивости получены для систем с одним входом и одним выходом, а также для многомерных систем. Численное моделирование подтверждает теоретические результаты и демонстрирует эффективность предложенного метода.

Разработанный подход расширяет геометрическую интерпретацию прямого метода Ляпунова и может быть использован при проектировании робастных систем управления с гарантированным апериодическим характером переходных процессов.

Ключевые слова: *апериодическая устойчивость; робастная устойчивость; метод градиентной скорости; функции Ляпунова; линейные системы управления; параметрическая неопределенность.*

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Received 30.03.2026
Revised 29.05.2026
Accepted 02.06.2026

https://doi.org/10.53360/2788-7995-2026-3(23)-2

BY 4.0
Check for updates

ҒТАХР: 10.23.01

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КРИПТОГРАФИЯЛЫҚ ХАТТАМА: ИНТЕРАКТИВТІ ЕМЕС НӨЛДІК ЖАРИЯ ЕТІЛГЕН ДӘЛЕЛДЕМЕЛЕРДІҢ ЕСЕПТЕУ КҮРДЕЛІЛІГІН ТАЛДАУ

Аңдатпа: *Бұл мақалада интерактивті емес нөлдік жария етілген дәлелдемелердің есептеу күрделілігін ресми тұрғыдан және асимптоталық талдау қарастырылады. Интерактивті емес нөлдік жария етілген дәлелдемелер дәлелдеушіге белгілі бір хабарламаның дұрыстығын, осы хабарламаны тексерушіге қосымша ақпараттарды жасырын дәлелдеуге мүмкіндік береді. Үйлестірілген жүйелерде жиі пайдаланылады. Зерттеуімізде интерактивті емес нөлдік жария етілген дәлелдемелердің сұлбаларының алғашқы базалық мәндер CRS моделында қарастырылады. Есептеу күрделілігін бағалауда негізігі базалық мәндер есебінде дәлелдеу және тексеру куәгердің өлшемі, дәлелдеу көлемі және алгоритмнің уақыт күрделілігі есептелінеді. Көпмүшелік есептеу моделі арқылы алгоритмдердің тиімділігі талданады. Тексеруші алгоритм күрделілігі және дәлелденушінің есептеу күрделілігі дәлдеменеің ұзындығымен анықталады. Дәлелдемелердің асимптоталық көрсеткіштер олардың қолданбалы нәтижелерге әсері зерттеледі. Нәтижелер негізінде интерактивті емес нөлдік жария етілген дәлелдемелердің айырмашылықтары мен ұқсастықтарын талдаудың әдіснамалық негізін және жоғары есептеу өнімділігі бар криптографиялық хаттамаларды жасауды қамтамасыз етеді.*

Талдау нәтижелері көрсеткендей алгоритмдердің есептеу күрделілігі куәгердің көрсеткішіне елеулі түрде тәуелдікті, ал дәлелдеудің ұзындығы тексеру алгоритмінің күрделілігін айқындайды. Бұға қоса, дәлелдеменің асимптоталық көлемдері олардың мәліметтер шығынына тигізетін ықпалы талданды. Осы алынған нәтижелер интерактивті емес нөлдік жария етілген дәлелдемелерді айырмашылықтары мен ұқсастықтарын талдау мен есептеу қолайлы криптографиялық хаттамаларды жасау үшін әдіснамалық негіз ретінде қолданылуы мүмкін.

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