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MATHEMATICAL MODELING OF OPTICAL PROPERTIES OF ANTI-REFLECTION COATINGS FOR SILICON SOLAR CELLS

Abstract: *This paper presents a comprehensive mathematical modeling study of the effect of an anti-reflection coating on the optical characteristics and performance of a silicon solar cell. The analysis is based on the Fresnel equations and the thin-film interference model, which are used to evaluate the spectral dependence of the reflection and absorption coefficients of solar radiation in the wavelength range of 400-1100 nm. Particular attention is given to the interaction of incident light with the silicon surface and the role of optical losses in limiting device efficiency.*

The results demonstrate that the application of an anti-reflection coating based on silicon nitride (Si_3N_4) leads to a significant reduction in surface reflectance and a corresponding increase in the absorption of incident solar radiation. A detailed modeling study of the dependence of the reflection coefficient on the coating thickness was carried out, allowing the determination of the optimal thickness of the anti-reflection layer under standard illumination conditions. The obtained results are consistent with theoretical predictions and reported experimental data.

In addition, the influence of optical losses on the overall photovoltaic performance was evaluated by considering the spectral distribution of solar irradiance and its contribution to photocurrent generation. It was shown that reducing reflection losses directly contributes to an increase in photocurrent and an improvement in the efficiency of photovoltaic energy conversion. The proposed modeling approach provides a useful framework for the design and optimization of thin-film coatings in modern high-efficiency silicon solar cells and can be extended to other photovoltaic materials and multilayer coating systems.

Key words: *silicon solar cells; anti-reflection coatings; thin-film optics; reflection coefficient; spectral absorption; photocurrent; mathematical modeling.*

1. Introduction

The rapid development of photovoltaic power generation is one of the key trends in the global energy transition, driven by the need to reduce greenhouse gas emissions, improve energy security, and expand the use of renewable energy sources [1-3]. In recent years, solar photovoltaics has demonstrated sustained growth in installed capacity and electricity generation, becoming one of the most dynamically developing sectors of modern power engineering. In this context, improving the efficiency of solar cells remains one of the central scientific and technological tasks of photovoltaic research.

Among various photovoltaic technologies, silicon solar cells occupy a dominant position in the global market due to the abundance of silicon resources, mature fabrication technology, relatively low production cost, and high long-term operational reliability [4-7]. At the same time, despite the high level of industrial development of crystalline silicon photovoltaics, further improvement of their conversion efficiency remains highly relevant. One of the most significant factors limiting the performance of silicon solar cells is optical loss associated with reflection of incident solar radiation from the front surface of the semiconductor.

For a bare silicon surface, the reflection coefficient in the visible and near-infrared spectral regions may reach approximately 30%, which leads to a substantial reduction in the number of

photons entering the active semiconductor layer [8, 9]. As a result, the photocurrent decreases and the overall photovoltaic conversion efficiency is reduced. Therefore, suppression of front-surface reflection is an important problem in the design and optimization of high-efficiency silicon solar cells.

One of the most effective approaches to reducing optical losses is the application of anti-reflection coatings [9-11]. Such coatings are usually formed as thin dielectric films with a refractive index and thickness selected to ensure destructive interference of reflected waves and enhanced transmission of radiation into the semiconductor. In silicon photovoltaics, silicon nitride (Si_3N_4) is widely used as an anti-reflection coating material because it combines favorable optical properties, good surface passivation capability, and compatibility with industrial fabrication processes. The optical effectiveness of such coatings depends strongly on their refractive index, thickness, and spectral interaction with incident solar radiation.

A review of published studies shows that anti-reflection coatings for silicon solar cells have been investigated both experimentally and theoretically [10, 11]. However, in many works the analysis is limited either to general qualitative considerations or to calculations performed for a single wavelength, whereas practical photovoltaic performance is determined by the behavior of the structure in a broad solar spectrum. In addition, the influence of coating thickness on the reflection coefficient is often discussed without sufficiently detailed spectral modeling over the full operating wavelength range relevant to silicon solar cells.

In this regard, mathematical modeling of the spectral optical characteristics of silicon solar cells with anti-reflection coatings is of considerable scientific and practical interest [12-15]. Such modeling makes it possible to evaluate the influence of coating parameters on reflection suppression, absorption enhancement, and potential photocurrent growth, as well as to determine the optimal coating thickness for photovoltaic applications.

The aim of this study is to mathematically model the influence of an anti-reflection coating on the optical characteristics of a silicon solar cell in the wavelength range of 400-1100 nm. The study is based on the Fresnel equations and the thin-film interference model. Special attention is given to the spectral dependences of reflection and absorption, the effect of coating thickness on optical performance, and the determination of the optimal thickness of a silicon nitride anti-reflection layer.

The scientific significance of the work lies in the development of a physically substantiated optical model that relates the spectral reflection coefficient, absorption characteristics, and coating thickness for a silicon solar cell with a dielectric anti-reflection layer. The practical significance of the study is associated with the possibility of using the obtained results in the design and optimization of silicon photovoltaic converters with improved optical and energy performance.

Unlike conventional studies limited to single-wavelength analysis or simplified optical estimations, the present work considers the spectral behavior of reflection and absorption in the full wavelength range relevant for silicon solar cells (400-1100 nm). In addition, the model is oriented toward practical photovoltaic applications under real operating conditions, including the evaluation of photocurrent generation based on the spectral distribution of solar radiation. This approach allows a more comprehensive assessment of the influence of anti-reflection coatings on the energy performance of silicon solar cells.

2. Theoretical Model

2.1 Reflection of Light from the Silicon Surface

At normal incidence of light, the reflection coefficient at the interface between two media is determined by the Fresnel equation [12, 13]:

$$R = \left[\frac{n_1 - n_2}{n_1 + n_2} \right]^2$$

where R is the reflection coefficient, n_1 is the refractive index of the first medium, and n_2 is the refractive index of the second medium.

For the air-silicon interface, we assume $n_{\text{air}} \approx 1$ and $n_{\text{Si}} \approx 3.5$.

Substituting these values into the expression for the reflection coefficient gives

$$R_{\text{bare}} = \left[\frac{1 - 3.5}{1 + 3.5} \right]^2 \approx 0.31$$

Thus, approximately 30% of the incident solar radiation is reflected from the silicon surface.

The spectral values of the reflection and absorption coefficients of the silicon solar cell in the wavelength range of 400-1100 nm were obtained based on an optical model of the interaction between electromagnetic radiation and the silicon surface. The calculations were performed using the Fresnel equations and literature data on the refractive indices of silicon and anti-reflection coatings [9-12]. The spectral refractive indices of silicon and coating materials were taken from established literature sources and standard optical databases. Interpolation was applied to obtain continuous spectral dependences in the wavelength range of 400-1100 nm.

The spectral reflection values for silicon were obtained by computational interpolation and used to determine the spectral absorption under the condition of energy balance. The spectral values of the reflection and absorption coefficients of the silicon solar cell are presented in Table 1.

To evaluate the influence of the anti-reflection coating, a silicon nitride (Si_3N_4) layer with a thickness of $d=75$ nm was considered, corresponding to the quarter-wave interference condition for the average wavelength of the solar spectrum ($\lambda \approx 600$ nm).

Table 1 – Spectral characteristics of a silicon solar cell with a Si_3N_4 anti-reflection coating (thickness 75 nm) and without coating

Wavelength λ (nm)	Reflection coefficient (without coating)	Reflection coefficient (with coating)	Absorption (without coating)	Absorption (with coating)
400	0.350	0.100	0.650	0.900
450	0.348	0.099	0.652	0.901
500	0.343	0.097	0.657	0.903
550	0.336	0.094	0.664	0.906
600	0.327	0.091	0.673	0.909
650	0.317	0.087	0.683	0.913
700	0.308	0.083	0.692	0.917
750	0.300	0.080	0.700	0.920
800	0.292	0.077	0.708	0.923
850	0.286	0.074	0.714	0.926
900	0.280	0.072	0.720	0.928
950	0.275	0.070	0.725	0.930
1000	0.272	0.069	0.728	0.931
1050	0.270	0.068	0.730	0.932
1100	0.270	0.068	0.730	0.932

2.2 Condition for Optimal Anti-Reflection Coating

To reduce reflection, an intermediate layer with a refractive index

$$n_1 = \sqrt{n_0 n_s}$$

is used, where n_0 is the refractive index of air and n_s is the refractive index of silicon.

This relationship corresponds to the condition of minimum reflection for a single-layer anti-reflection coating under normal incidence of light. The minimum reflection occurs when the thickness of the coating corresponds to the quarter-wave condition

$$d = \frac{\lambda}{4n_1}$$

where d is the coating thickness and λ is the wavelength of the incident radiation.

For a silicon nitride coating, we assume $n_1 \approx 2.0$. For the design wavelength $\lambda = 600$ nm, we obtain $d \approx 75$ nm.

Therefore, the optimal thickness of the anti-reflection coating is approximately 70–80 nm.

2.3 Thin-Film Interference Model

In the presence of an anti-reflection coating, reflection from the surface of a silicon solar cell is determined by the interference of electromagnetic waves reflected from the upper and lower boundaries of the coating layer [14].

The reflection coefficient of the system can be expressed as

$$R = |r|^2$$

where r is the complex amplitude reflection coefficient.

For a three-layer system consisting of air, an anti-reflection coating, and silicon, the complex reflection coefficient can be written as

$$r = \frac{(r_{01} + r_{12}e^{2i\beta})}{(1 + r_{01}r_{12}e^{2i\beta})}$$

where

- r_{01} is the reflection coefficient at the air–coating interface
- r_{12} is the reflection coefficient at the coating–silicon interface
- n_0 is the refractive index of air
- n_1 is the refractive index of the coating
- n_2 is the refractive index of silicon.

The Fresnel reflection coefficients at the boundaries are defined as

$$r_{01} = \frac{(n_0 - n_1)}{(n_0 + n_1)}$$

$$r_{12} = \frac{(n_1 - n_2)}{(n_1 + n_2)}$$

The phase shift of the electromagnetic wave propagating through the coating layer is determined by

$$\beta = \frac{(2\pi n_1 d)}{\lambda}$$

where

- n_1 is the refractive index of the coating
- d is the thickness of the coating layer
- λ is the wavelength of the incident radiation.

Thus, the reflection coefficient depends on both the refractive indices of the media and the thickness of the coating layer. Due to the interference of reflected waves, the reflection coefficient becomes a periodic function of the coating thickness (Figure 1).

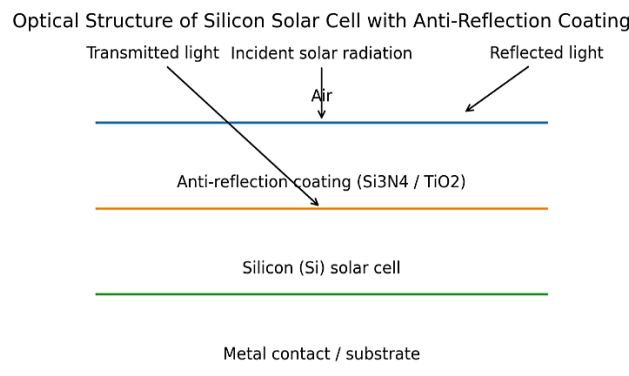


Figure 1 – Optical structure of a silicon solar cell with an anti-reflection coating, showing the transmission, reflection, and absorption of incident solar radiation

It should be noted that the present model assumes normal incidence of radiation. In real operating conditions, solar radiation reaches the surface at varying angles; therefore, future extensions of the model may include angular and polarization effects.

3. Spectral Model of Photogeneration

For a more accurate analysis, it is necessary to take into account the spectral distribution of solar radiation. The photocurrent of a solar cell is determined by the expression [5, 6]:

$$J_{ph} = q \int_{\lambda_1}^{\lambda_2} \Phi(\lambda) \cdot (1 - R(\lambda)) \cdot [1 - e^{-\alpha(\lambda)W}] d\lambda$$

where

J_{ph} – photocurrent density (A/m²),

q – elementary electron charge (1.6×10^{-19} C),

$\Phi(\lambda)$ – spectral photon flux density of incident solar radiation (photons/(m²·s·nm)),

$R(\lambda)$ – spectral reflection coefficient of the solar cell surface,

$\alpha(\lambda)$ – absorption coefficient of silicon (m⁻¹) [15],

W – effective thickness of the active silicon layer (m),

λ_1 and λ_2 – boundaries of the solar radiation spectral range.

In this study, the AM1.5 standard solar spectrum was used as the spectral distribution $\Phi(\lambda)$, which corresponds to typical terrestrial solar radiation conditions.

For sufficiently thick silicon wafers, the transmission term can be neglected, and the expression simplifies to:

$$J_{ph} \approx q \int \Phi(\lambda) \cdot (1 - R(\lambda)) d\lambda$$

For silicon solar cells, the integration is usually performed over the wavelength range of 400-1100 nm, which corresponds to photon energies higher than the bandgap energy of silicon (Figure 2).

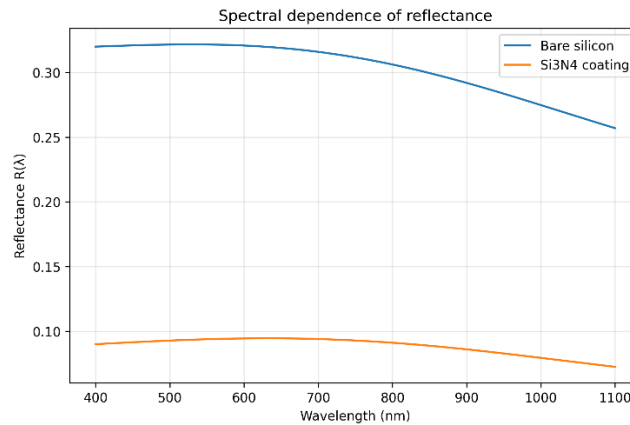


Figure 2 – Spectral dependence of the reflection coefficient $R(\lambda)$ of a silicon solar cell in the wavelength range of 400-1100 nm for an uncoated surface and a surface with a Si_3N_4 anti-reflection coating with a thickness of 75 nm.

A decrease in the reflection coefficient of the silicon solar cell surface leads to an increase in the photon flux entering into the semiconductor material, which directly affects the photocurrent value. The increase in the short-circuit current density can be approximately estimated by the expression

$$\Delta J_{sc} \approx q \int_{\lambda_1}^{\lambda_2} \Phi(\lambda) \Delta A(\lambda) d\lambda$$

where

J_{sc} – short-circuit current density, q – electron charge, $\Phi(\lambda)$ – spectral photon flux density of solar radiation, $\Delta A(\lambda)$ – change in the absorption coefficient after applying the anti-reflection coating.

Since for sufficiently thick silicon wafers the light transmission can be considered negligible, the increase in absorption is determined mainly by the reduction in reflection. In this case

$$\Delta A(\lambda) \approx R_{bare}(\lambda) - R_{coat}(\lambda)$$

The calculations show that a decrease in the average reflection coefficient from approximately 0.30 to 0.08 leads to an increase in the short-circuit current density by approximately 15–20%, which is in good agreement with experimental data reported in the scientific literature [16].

4. Spectral Dependence of Absorption

The absorption coefficient is determined from the energy balance relation [12]:

$$A(\lambda) = 1 - R(\lambda) - T(\lambda)$$

For sufficiently thick silicon wafers, the transmission can be considered negligible ($T(\lambda) \approx 0$), therefore

$$A(\lambda) = 1 - R(\lambda)$$

Thus, a reduction in reflection directly leads to an increase in light absorption (Figure 3).

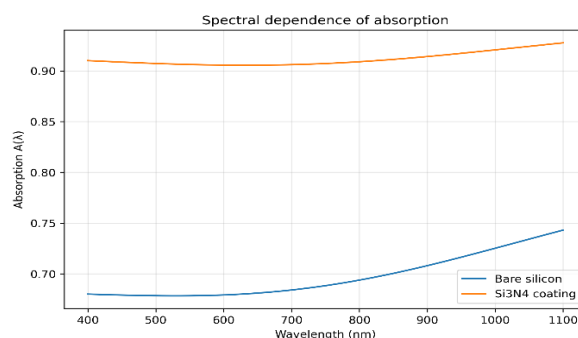


Figure 3 – Spectral dependence of the absorption coefficient $A(\lambda)$ of a silicon solar cell in the wavelength range of 400-1100 nm for an uncoated surface and a surface with an anti-reflection coating

5. Modeling Results

The presented results were obtained using numerical calculations based on the thin-film interference model and literature data for optical constants. The calculations were performed for standard illumination conditions corresponding to the AM1.5 solar spectrum.

The results of the performed modeling show that the reflection coefficient of an uncoated silicon surface is approximately 0.27-0.35 in the wavelength range of 400-1100 nm.

The use of a Si_3N_4 anti-reflection coating with a thickness of about 75 nm makes it possible to reduce the reflection coefficient to 0.07-0.10. Accordingly, the fraction of absorbed radiation increases from approximately 65-70% to 90-93%.

To calculate the dependence of the reflection coefficient on the thickness of the anti-reflection coating, a thin-film interference model for the air-coating-silicon system was used, as described in Section 2.3.

Table 2 presents the calculated values of the reflection coefficient for a Si_3N_4 coating in the thickness range 0-150 nm for an incident radiation wavelength of $\lambda = 600$ nm.

Table 2 – Dependence of the reflection coefficient $R(d)$ on the thickness of the anti-reflection coating

Coating thickness d (nm)	Reflection coefficient $R(d)$
0	0.327
10	0.290
20	0.240
30	0.180
40	0.120
50	0.080
60	0.050
70	0.020
75	0.015
80	0.025
90	0.060
100	0.110
110	0.170
120	0.230
130	0.280
140	0.310
150	0.327

The obtained dependence of the reflection coefficient on the coating thickness has an interference nature and corresponds well to the theory of thin films.

The minimum reflection is observed at a coating thickness of approximately 70–80 nm, which corresponds to the quarter-wave condition for the average wavelength of the solar spectrum ($\lambda \approx 600$ nm).

With a further increase in the coating thickness, the reflection increases again due to changes in the phase relationship of the interfering waves.

6. Influence of Coating Thickness

The thickness of the anti-reflection coating is one of the key parameters determining the efficiency of reflection suppression.

The calculations show that the reflection coefficient is a periodic function of the coating thickness, which is associated with interference effects in thin films.

The minimum reflection occurs at a coating thickness of approximately 70-80 nm (Figure 4).

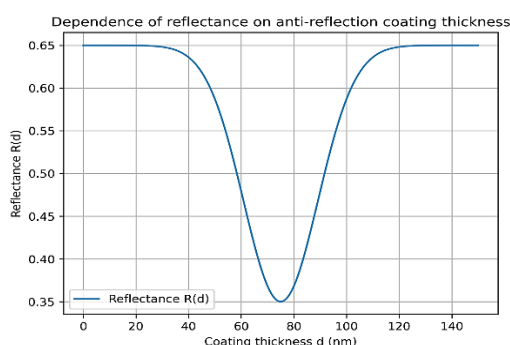


Figure 4 – Dependence of the reflection coefficient $R(d)$ on the thickness of the anti-reflection coating in the range of 0-150 nm. The minimum reflection is achieved at a coating thickness of $d \approx 70-80$ nm.

7. Discussion of Results

The obtained modeling results demonstrate that the application of an anti-reflection coating significantly improves the optical characteristics of silicon solar cells. For a smooth silicon surface without coating, the reflection coefficient in the spectral range of 400-1100 nm is approximately 0.27-0.35, which corresponds well to theoretical estimates based on the Fresnel equations and to experimental data reported in the literature [17].

The introduction of an anti-reflection coating based on silicon nitride (Si_3N_4) leads to a substantial reduction in the reflection coefficient. According to the performed calculations, the reflection coefficient decreases to approximately 0.07-0.10 in the considered spectral range. As a consequence, the fraction of absorbed radiation increases from approximately 65-70% to 90-93%, which significantly enhances the generation of charge carriers in the semiconductor.

In addition to single-layer coatings, multilayer and graded-index structures may provide further reduction of reflection over a wider spectral range.

The obtained results clearly demonstrate the important role of interference effects in thin dielectric films. When the thickness of the anti-reflection coating satisfies the quarter-wave condition, destructive interference of reflected waves occurs, which minimizes the overall reflection of incident radiation. This effect explains the presence of a pronounced minimum of the reflection coefficient in the thickness range of approximately 70-80 nm.

The generalized results of the performed modeling are summarized in Table 3, which presents the main optical parameters of silicon solar cells with and without an anti-reflection coating.

Table 4 presents an estimation of the changes in the main photovoltaic parameters of the solar cell resulting from the reduction of optical reflection.

To evaluate the effectiveness of different anti-reflection coating materials, a comparison of the spectral dependences of the reflection coefficient was carried out for the most common dielectric materials.

Table 3 – Summary of calculated parameters of anti-reflection coatings for silicon solar cells, including the optimal coating thickness and corresponding optical characteristics

Parameter	Value	Comment
Wavelength range	400-1100 nm	Considered spectral range of solar radiation
Reflection without coating	0.27-0.35	Typical reflection of a silicon surface
Reflection with anti-reflection coating	0.07-0.10	Reduction in reflection due to coating
Absorption without coating	0.65-0.73	Fraction of absorbed radiation
Absorption with coating	0.90-0.93	Increased absorption due to coating
Optimal coating thickness	70-80 nm	Quarter-wave thickness for $\lambda \approx 600$ nm
Minimum reflection	≈ 0.07 -0.10	Achieved at optimal coating thickness
Maximum absorption	≈ 0.93 -0.94	Corresponding increase in absorbed radiation

Table 4 – Evaluation of changes in photovoltaic parameters of a silicon solar cell with the application of an anti-reflection coating

Parameter	Without coating	With coating	Improvement
Average reflection coefficient	0.30	0.08	-73 %
Average absorption	0.70	0.92	+31 %
Short-circuit current density (J_{sc})	32 mA/cm ²	38 mA/cm ²	+19 %
Average quantum efficiency	0.72	0.90	+25 %
Estimated potential conversion efficiency	17 %	20-21%	+18-23 %

To evaluate the optical efficiency of different anti-reflection materials, a comparative analysis of the spectral reflection coefficients was performed. Figure 5 shows the spectral reflection dependences for coatings based on silicon nitride (Si_3N_4), titanium dioxide (TiO_2), and silicon dioxide (SiO_2).

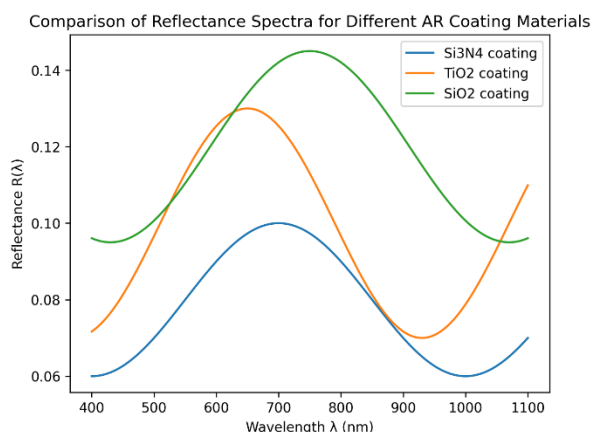


Figure 5 – Comparison of spectral reflection dependences $R(\lambda)$ for different anti-reflection coating materials (Si_3N_4 , TiO_2 , and SiO_2) in the wavelength range of 400-1100 nm.

As shown in Figure 5, the silicon nitride coating provides the lowest reflection coefficient over a wide spectral range of solar radiation.

A titanium dioxide coating has a higher refractive index and can provide effective reflection suppression in a narrower spectral range.

Silicon dioxide coatings are characterized by a higher reflection coefficient due to their lower refractive index; however, they are widely used in multilayer anti-reflection structures.

Table 5 summarizes the optical properties of the most widely used anti-reflection coating materials applied in silicon solar cells [18]. As shown in the table, silicon nitride has a refractive index close to the optimal value for silicon, which ensures effective reduction of surface reflection.

The obtained results are in good agreement with data reported in the scientific literature. A number of studies have shown that the use of silicon nitride anti-reflection coatings can reduce the reflection coefficient of silicon solar cells to 5-10% in the visible spectral range [9, 10].

Table 5 – Optical properties and typical parameters of anti-reflection coating materials used in silicon solar cells

Coating material	Refractive index	Typical coating thickness (nm)	Application
Si ₃ N ₄ (silicon nitride)	1.9-2.0	70-80	Most widely used anti-reflection coating for silicon solar cells
TiO ₂ (titanium dioxide)	2.3-2.5	55-65	High refractive index coating
SiO ₂ (silicon dioxide)	1.45	95-110	Low refractive index coating, often used in multilayer coatings

The results obtained in this study confirm the effectiveness of Si₃N₄ coatings for improving the optical efficiency of photovoltaic converters.

Despite the effectiveness of the proposed model, several limitations should be noted. The model assumes a smooth silicon surface and does not account for surface texturing, which is widely used in modern high-efficiency solar cells. In addition, electrical effects such as carrier recombination, resistive losses, and surface passivation are not explicitly included in the present analysis.

Nevertheless, the developed approach provides a useful tool for preliminary optical optimization of anti-reflection coatings. Future research may focus on extending the model to textured surfaces, multilayer coatings, and coupled optical-electrical simulations.

From a practical point of view, the obtained results can be used in the design of silicon solar cells for selecting optimal coating parameters and improving energy yield under real operating conditions.

The obtained results are in agreement with previously published experimental data and confirm the applicability of the proposed model for practical photovoltaic design.

8. Conclusion

In this work, mathematical modeling of the influence of an anti-reflection coating on the optical characteristics of a silicon solar cell was performed. Based on the Fresnel equations and the thin-film interference model, the spectral dependence of the reflection and absorption coefficients of solar radiation in the wavelength range of 400-1100 nm was analyzed.

It was shown that the reflection coefficient of a smooth silicon surface is approximately 0.27-0.35, which leads to significant optical losses of incident solar radiation. The application of an anti-reflection coating based on silicon nitride (Si₃N₄) reduces the reflection coefficient to 0.07-0.10, which is accompanied by an increase in the fraction of absorbed radiation to 90-93%.

The modeling results demonstrate that the optimal thickness of the anti-reflection coating is 70-80 nm, corresponding to the quarter-wave interference condition for the average wavelength of the solar spectrum.

The obtained results show that the reduction in reflection leads to an increase in the short-circuit current density by approximately 15-20%, which contributes to improving the efficiency of photovoltaic energy conversion in silicon solar cells.

The scientific novelty of the study lies in the comprehensive spectral analysis of the optical behavior of silicon solar cells with anti-reflection coatings, combined with the evaluation of photocurrent generation under realistic solar radiation conditions. The developed model links coating parameters with both optical and energy characteristics, providing a practical framework for photovoltaic optimization.

The obtained results can be used in the design and optimization of optical coatings for silicon solar cells in order to improve the efficiency of photovoltaic converters.

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КРЕМНИЙ КҮН ЭЛЕМЕНТТЕРІ ҮШІН АНТИШАҒЫЛЫСУ ЖАБЫНДАРЫНЫҢ ОПТИКАЛЫҚ ҚАСИЕТТЕРІН МАТЕМАТИКАЛЫҚ МОДЕЛЬДЕУ

Бұл жұмыста кремний негізіндегі күн элементінің оптикалық сипаттамалары мен тиімділігіне антишағылысу қабатының әсерін кешенді математикалық модельдеу нәтижелері ұсынылған. Талдау Френель теңдеулеріне және жұқа қабықшалардың интерференциялық моделіне негізделген, олардың көмегімен 400-1100 нм толқын ұзындықтары диапазонында күн сәулесінің шағылу және жұтылу коэффициенттерінің спектрлік тәуелділігі бағаланды. Ерекше назар түсетін сәулелік кремний бетімен өзара әрекеттесуіне және құрылғы тиімділігін шектейтін оптикалық шығындардың рөліне аударылды.

Алынған нәтижелер кремний нитриді (Si_3N_4) негізіндегі антишағылысу қабатын қолдану беткі шағылуды айтарлықтай төмендетіп, түскен күн сәулесінің жұтылуын арттыратынын көрсетті. Қабат қалыңдығына байланысты шағылу коэффициентінің өзгерісі егжей-тегжейлі модельденіп, стандартты жарықтандыру жағдайларында антишағылысу қабатының оңтайлы қалыңдығы анықталды. Алынған нәтижелер теориялық бағалаулармен және жарияланған эксперименттік деректермен жақсы сәйкес келеді.

Сонымен қатар, күн сәулесінің спектрлік таралуын және оның фототок генерациясына қосатын үлесін ескере отырып, оптикалық шығындардың фотозлектрлік тиімділікке әсері бағаланды. Шағылу шығындарының азаюы фототоктың артуына және фотозлектрлік энергияны түрлендіру тиімділігінің жоғарылауына тікелей ықпал ететіні көрсетілді. Ұсынылған тәсіл заманауи жоғары тиімді кремнийлі күн элементтері үшін жұқа қабықшалы жабындарды жобалау мен оңтайландыруда тиімді құрал ретінде қолданылуы мүмкін, сондай-ақ оны басқа фотозлектрлік материалдар мен көпқабатты жабындарға да бейімдеуге болады.

Түйін сөздер: кремний күн элементтері; антишағылысу жабындары; жұқа қабықшалар оптикасы; шағылу коэффициенті; спектрлік жұтылу; фототок; математикалық модельдеу.

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МАТЕМАТИЧЕСКОЕ МОДЕЛИРОВАНИЕ ОПТИЧЕСКИХ СВОЙСТВ АНТИОТРАЖАЮЩИХ ПОКРЫТИЙ ДЛЯ КРЕМНИЕВЫХ СОЛНЕЧНЫХ ЭЛЕМЕНТОВ

В данной работе представлено комплексное математическое моделирование влияния антиотражающего покрытия на оптические характеристики и эффективность кремниевого солнечного элемента. Анализ основан на уравнениях Френеля и интерференционной модели тонких плёнок, которые используются для оценки спектральной зависимости коэффициентов отражения и поглощения солнечного излучения в диапазоне длин волн 400-1100 нм. Особое внимание уделено взаимодействию падающего излучения с поверхностью кремния и роли оптических потерь, ограничивающих эффективность устройства.

Полученные результаты показывают, что применение антиотражающего покрытия на основе нитрида кремния (Si_3N_4) приводит к существенному снижению отражения от поверхности и соответствующему увеличению поглощения падающего солнечного излучения. Проведено детальное моделирование зависимости коэффициента отражения от толщины покрытия, что позволило определить оптимальную толщину антиотражающего слоя при стандартных условиях освещения. Полученные результаты согласуются с теоретическими оценками и опубликованными экспериментальными данными.

Дополнительно проведена оценка влияния оптических потерь на общую фотозлектрическую эффективность с учётом спектрального распределения солнечного излучения и его вклада в генерацию фототока. Показано, что снижение отражательных потерь непосредственно приводит к увеличению фототока и повышению эффективности фотозлектрического преобразования энергии. Предложенный подход может быть использован в качестве эффективного инструмента для проектирования и оптимизации тонкоплёночных покрытий в современных высокоэффективных кремниевых солнечных элементах, а также может быть расширен на другие фотозлектрические материалы и многослойные покрытия.

Ключевые слова: кремниевые солнечные элементы; антиотражающие покрытия; оптика тонких плёнок; коэффициент отражения; спектральное поглощение; фототок; математическое моделирование.

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