Fariza Bilyalovna Tebuyeva – Doctor of Physical and Mathematical Sciences, Professor of the Department of Computational Mathematics and Cybernetics, North Caucasus Federal University, Stavropol, Russian Federation; e-mail: ftebueva@ncfu.ru. ORCID: https://orcid.org/0000-0001-9324-9259.

Nazym Zhumageldyevna Mukusheva* – PhD student (Doctoral student) of the Department of Information Security, L.N. Gumilyov Eurasian National University, Astana, Kazakhstan; e-mail: nazym9_1@mail.ru ORCID: https://orcid.org/0009-0001-5015-3617.

Редакцияға енуі 06.01.2026
Өңдеуден кейін түсуі 20.02.2026
Жариялауға қабылданды 23.02.2026

https://doi.org/10.53360/2788-7995-2026-1(21)-13

IRSTI: 55.47.03

Z.A. Kutpanova[1], D.O. Kozhakhmetova[2*], K.M. Suleymenov[3], G.N. Baiseitov[4], D.N. Shandronov[5]

[1]Astana IT University, 010000, Kazakhstan, Astana,Mangilik El, 55/11
[2]Shakarim University,
071412, Kazakhstanst, Semey, Glinka, 20 A
[3]L.N. Gumilyov Eurasian National University,
10000, Kazakhstan, Astana, Stepaeva 2
[4]R&D Center «Kazakhstan Engineering»,
010000, Kazakhstan Astana, Auezov Str. 2
[5]Military Engineering Institute of Radio Electronics and Communications,
Almaty, Kazakhstan, Zandosova, 53
*e-mail: d.kojahmetova@shakarim.kz

OPTIMIZATION OF FLIGHT TRAJECTORIES OF UNMANNED AERIAL VEHICLES WITH GIVEN FINAL CONDITIONS

Abstract: *The article considers the urgent problem of optimizing the flight trajectory of unmanned aerial vehicles (UAVs) with given final conditions. An approach to constructing the optimal UAV trajectory along successive sections using approximation by a fifth-degree polynomial is presented. A mathematical model is developed that takes into account the features of using local inertial coordinate systems on each section of the trajectory. A motion optimization method is proposed based on minimizing the quadratic functional characterizing both the accuracy of reaching specified points and the energy costs for control. The solution to the optimization problem is obtained using analytical design of the optimal controller. Analytical expressions are given for determining the optimal lateral acceleration of the UAV, ensuring the required positioning accuracy with minimal energy costs. Features of coordinate transformation between trajectory sections to ensure continuity of motion are considered. A practical example of calculating the optimal UAV trajectory with given initial conditions is given.*

Key words: *unmanned aerial vehicle, trajectory optimization, polynomial approximation, inertial coordinate systems, quadratic functional, optimal control, segmented trajectory*.

Introduction

Unmanned aerial vehicles (UAVs) are increasingly required to execute complex multi-stage missions under strict constraints on positioning accuracy, timing, and energy consumption. Typical missions include area reconnaissance, payload delivery, monitoring tasks, and return-to-base operations, each imposing specific kinematic and dynamic requirements. The successful execution of such missions critically depends on the quality of trajectory planning and control algorithms.

Trajectory planning for UAVs must account for physical and operational constraints, including flight dynamics, actuator limitations, time restrictions, and safety margins. In practical applications, inefficient trajectory generation may lead to excessive energy consumption or violation of mission constraints, resulting in incomplete task execution. These challenges become particularly pronounced in low-altitude environments, where flight paths must satisfy altitude limits, obstacle clearance, and predefined airspace corridors [1, 2].

Mission-critical applications such as search and rescue, medical logistics, and inspection tasks require deterministic and predictable trajectory behavior. In such scenarios, planning algorithms must ensure not only optimality with respect to a selected performance criterion, but also robustness and reproducibility of the generated trajectories. This motivates the use of deterministic trajectory planning methods that provide explicit analytical descriptions of motion and allow direct incorporation of boundary and continuity conditions.

Existing UAV trajectory planning approaches can be broadly classified into deterministic analytical methods and numerical or heuristic optimization techniques. Classical deterministic methods are based on variational calculus and optimal control theory, with Pontryagin's maximum principle serving as a fundamental tool for deriving optimal control laws for constrained dynamical systems. These methods provide mathematically rigorous solutions and allow analytical characterization of system behavior, but often suffer from high computational complexity and sensitivity to modeling accuracy, especially in high-dimensional problems. While numerical and heuristic methods, including evolutionary and swarm intelligence algorithms, offer flexibility, they generally lack guarantees of smoothness, continuity, and predictability, limiting their applicability in precision-critical missions.

In this context, the present study addresses the problem of deterministic UAV trajectory planning using a segment-wise, polynomial-based representation within local inertial coordinate frames**.** By employing fifth-degree polynomial approximations for each trajectory segment, the method enables analytical computation of position, velocity, and acceleration, while simultaneously allowing synthesis of optimal control laws with reduced computational complexity. The primary objective of the work is to develop a trajectory planning framework that guarantees smooth, feasible, and energy-efficient UAV motion along complex multi-stage paths, providing both high accuracy and practical implementability in real-time onboard systems [3-5].

Materials and methods

This paper considers the problem of optimizing the movement of an unmanned aerial vehicle (UAV) along a given trajectory while ensuring the required final conditions. The main task is to develop a method for forming an optimal UAV flight trajectory that ensures passage through specified points in space while minimizing energy consumption and ensuring the required positioning accuracy.

Mathematically, the problem is formulated as determining the optimal control of the UAV, ensuring movement along a trajectory passing through specified points in space with coordinates $(x^{(k)}, y^{(k)}, z^{(k)})$in the inertial reference system $(OXYZ)$, where k is the point number. It is necessary to minimize the functional characterizing both the accuracy of reaching the specified points and the energy costs for control.

Let us consider the problem of UAV movement along a given trajectory – the UAV flight is carried out along three successively given trajectories, $\gamma_1(t_0,t_1)$, $\gamma_2(t_1,t_2)$, $\gamma_3(t_2,t_3)$, where we will consider, $\gamma_1(t_0,t_1)$ – the initial interval, $\gamma_2(t_1,t_2)$ – the main flight interval, and $\gamma_3(t_2,t_3)$ – the interval including the return of the UAV to the starting point. In turn, $(t_0,t_3)$is the interval of the full flight, i.e. denotes the beginning and end of the flight. For a number of sections of at least 3, the problem will be solved similarly.

Methods of analytical synthesis of tasks and optimization of UAV control systems for flight along successive sections are based on the approaches of the theory of optimal control taking into account real conditions of application. The development begins with the creation of a mathematical model of the UAV flight trajectory. To describe the trajectory, an approximation by a polynomial of the appropriate type is used, which ensures an effective solution to control optimization problems [6-9].

$$\gamma(t) = \sum_{i=0}^{n} C_i t^i. \quad (1)$$

Here $\gamma(t)$is the change in spatial coordinates, $t$is the flight time of the UAV, in this case $t_0 \leq t \leq t_3$, and $C_i, i = \overline{1,n}$are the coefficients to be determined.

The polynomial is used to approximate the projection of the UAV trajectory onto each of the axes of the inertial reference system.

$$P_5(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3 + C_4 t^4 + C_5 t^5, \qquad (2)$$

1. To uniquely define a polynomial, it is necessary to specify six conditions (according to the number of coefficients $C_0, C_1, C_2, C_3, C_4, C_5$). These conditions may include the position, speed, and acceleration of the UAV at the initial and final points of the trajectory.

2. The fifth-degree polynomial ensures the continuity of not only the position function itself, but also its first and second derivatives, which corresponds to the continuity of velocity and acceleration. This is critically important for the feasibility of the trajectory by a real UAV, taking into account its dynamic characteristics.

3. A polynomial of degree less than 5 does not provide sufficient flexibility to simultaneously satisfy conditions for position, velocity, and acceleration at boundary points.

4. Polynomials of higher degrees can lead to undesirable oscillations of the trajectory between given points, which complicates control and increases energy costs.

5. The fifth-degree polynomial provides a compromise between the accuracy of approximation and the computational complexity of trajectory planning algorithms.

Polynomial (2) is used to approximate the projections of the UAV trajectory onto each of the axes of the inertial reference system. Thus, the spatial trajectory of the UAV is represented by three independent polynomials:

$$x(t) = C_0 + C_1 t + C_2 t^2 + C_3 t^3 + C_4 t^4 + C_5 t^5$$
$$y(t) = C_{0y} + C_{1y} t + C_{2y} t^2 + C_{3y} t^3 + C_{4y} t^4 + C_{5y} t^5$$
$$z(t) = C_{0z} + C_{1z} t + C_{2z} t^2 + C_{3z} t^3 + C_{4z} t^4 + C_{5z} t^5.$$

where $x(t), y(t), z(t)$are the coordinates in the inertial reference frame at the moment of time t.

To determine the coefficients of the polynomials, it is necessary to set the boundary conditions for each section of the trajectory. Such conditions include:

1. Coordinates of the UAV at the starting and ending points of the section:

$$\{x(t_0) = x_0, y(t_0) = y_0, z(t_0) = z_0.$$

2. Velocity projections at the initial and final points:

$$\{\dot{x}(t_0) = v_{x0}, \quad \dot{x}(t_f) = v_{xf}, \dot{y}(t_0) = v_{y0}, \quad \dot{y}(t_f) = v_{yf}, \dot{z}(t_0) = v_{z0}, \quad \dot{z}(t_f) = v_{zf}.$$

3. Acceleration projections at the initial and final points (if necessary):

$$\{\ddot{x}(t_0) = a_{x0}, \quad \ddot{x}(t_f) = a_{xf}, \ddot{y}(t_0) = a_{y0}, \quad \ddot{y}(t_f) = a_{yf}, \ddot{z}(t_0) = a_{z0}, \quad \ddot{z}(t_f) = a_{zf}.$$

Defining all these conditions allows us to uniquely calculate the coefficients of the polynomials for each coordinate.

To determine the kinematic characteristics of the UAV motion, it is necessary to find the derivatives of the polynomial function (2). The first derivative with respect to time will give an expression for the speed, and the second for the acceleration.

Thus, the fifth-degree polynomial model provides an effective representation of the UAV flight trajectory, allowing the analytical determination of the position, speed and acceleration of the device at any time along the trajectory, which is the basis for constructing an optimal control system.

Results and discussion

The axes of the inertial coordinate system are the lines that describe the position of the UAV in space and time in the inertial reference frame.

An inertial reference frame is a system in which the principle of inertia is preserved, i.e. if no external forces act on the UAV, it will move at a constant speed or remain at rest.

In practice, pairwise orthogonal lines are considered as the axes of the inertial system; for example, in three-dimensional space, X, Y, Zaxes designated as , ko are used, which determine the location of the UAV in space and describe the movement of the UAV.

Let us differentiate the polynomial from (2) twice, then

$$\dot{P}_5(t) = C_1 + 2C_2 t + 3C_3 t^2 + 4C_4 t^3 + 5C_5 t^4, \qquad (3)$$

$$\ddot{P}_5(t) = 2C_2 + 6C_3 t + 12C_4 t^2 + 20C_5 t^3, \qquad (4)$$

Equations (3) and (4) define the projections of velocity and acceleration. The coefficients $(C_0, \dots, C_5)$are determined by solving the system of equations for polynomials $\left(P_5(t), \dot{P}_5(t), \ddot{P}_5(t)\right)$for the end of the guidance. These expressions (3), (4) are used to calculate the projections of the UAV velocity and acceleration on the axes of the inertial coordinate system:

$$\{v_{x(t)} = C_{1x} + 2C_{2x}t + 3C_{3x}t^2 + 4C_{4x}t^3 + 5C_{5x}t^4, v_{y(t)} = C_{1y} + 2C_{2y}t + 3C_{3y}t^2 + 4C_{4y}t^3 + 5C_{5y}t^4, v_{z(t)} = C_{1z} + 2C_{2z}t + 3C_{3z}t^2 + 4C_{4z}t^3 + 5C_{5z}t^4. \quad (11)$$

Also

$$\{a_{x(t)} = 2C_{2x} + 6C_{3x}t + 12C_{4x}t^2 + 20C_{5x}t^3, a_{y(t)} = 2C_{2y} + 6C_{3y}t + 12C_{4y}t^2 + 20C_{5y}t^3, a_{z(t)} = 2C_{2z} + 6C_{3z}t + 12C_{4z}t^2 + 20C_{5z}t^3 \quad (5)$$

The coefficients C_1, C_2, C_3, C_4, C_5 for each coordinate axis are determined by solving a system of equations formed on the basis of boundary conditions. This allows us to obtain a trajectory that not only passes through the specified points in space, but also provides the required values of speed and acceleration at these points.

A common application task is the flight of a UAV along a trajectory passing through specified points in space with specified coordinates $(x^{(k)}, y^{(k)}, z^{(k)})$in an inertial reference system $(OXYZ)$, where k – indicates the number of the point through which the trajectory will pass.

Let us consider the problem of determining the trajectory of the UAV, where these sections are consecutive, on each of which the guidance of the optimal UAV flight is determined, and the requirements of the control system (CS) will also be met. The specified trajectory must pass with a given accuracy approximately to the specified points, and also ensure the task of minimizing the integral losses associated with the maneuvers of the UAV movement and the change in control overload. It is necessary to form optimality criteria, including the following flight components:

1. characteristics of the accuracy of achieving the set goal, reaching the final position of the UAV;
2. characteristic of integral losses during the control period.

This problem is solved by the classical Boltz problem of minimizing a functional of the form:

$$J = g(x_k, t_k) + \int_{t_0}^{t_k} F(x, t)dt, \quad (6)$$

Here $g(x_k, t_k)$ describes the final goal of the state $x(t)$ on the section $[t_0, t_k]$, and $\int_{t_0}^{t_k} F(X, t)dt$defines the integral losses in controlling the UAV movement. Note that the function $g(x_k, t_k)$defines the final goal of the UAV flight section between the specified k, namely, between the initial and final points, including the internal points of the UAV movement trajectory. Consequently, the function $g(x_k, t_k)$will characterize the minimum deviation of the UAV flight trajectory from the points $x_k, k = \underline{1, N}$, where $N$is the number of intermediate predetermined points of the UAV movement route.

To determine the trajectory of the UAV, a problem similar to the problems of rigid body kinematics is considered, determining the movement of the center of mass, thus, functional (5) is transformed into a quadratic functional [10-12].

$$J = X_k^T R X_k + \int_{t_0}^{t_k} [X(t)^T Q(t) X(t) + U(t)^T S(t) U(t)]dt, \quad (7)$$

Here $X_k = [\Delta x_k, \Delta y_k, \Delta z_k]^T$is the vector of minimum deviations relative to the intermediate point of the route, $X(t) = [x(t), y(t), z(t)]^T$is the vector of coordinates of the center of mass of the UAV, $U(t) = \left[a_x(t), a_y(t), a_z(t)\right]^T$is the control vector of normal accelerations of the center of mass.

To solve the problem of optimizing the functional (7), it is necessary to solve the problem of choosing matrices $R, Q(t), S(t)$. The functional (7) includes variable terms of different dimensions and values $X_k, X(t), U(t)$, so it is necessary to transform them into dimensionless quantities by normalizing using matrices $R, Q(t), S(t)$, and the admissible values of the variables $X_k, X(t), U(t)$included in the functional (6) were taken as normalizing coefficients.

To determine the optimal control U(t), it is necessary to solve the boundary value problem, i.e. to simultaneously solve the system of differential equations (5) and (7). In this case, in system (6), the specified conditions are the initial conditions, and in system (7), the specified conditions are the final conditions. The problem can be solved by the sweep method, in which, at the initial stage, approximate boundary values are specified for one of the systems (5) or (7), and then systems (5) and (7) are repeatedly integrated in direct and reverse time until the specified accuracy is achieved.

The kinematics of the UAV guidance scheme to a given point $\left(O^{(k)}, X^{(k)}, Y^{(k)}, Z^{(k)}\right)$ of the inertial reference system is shown in Fig. 1, the ordinal number of the point in space through which the UAV will move.

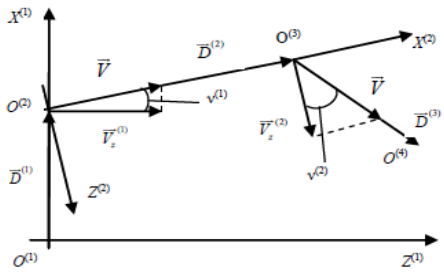

Figure 1 – UAV guidance

New inertial coordinate system $\left(O^{(k)}, X^{(k)}, Y^{(k)}, Z^{(k)}\right)$. For each segment of the UAV flight, the origin of the coordinate system $O^{(k)}$will coincide with the starting point of the trajectory. Then the axis $O^{(k)}X^{(k)}$indicates the direction of movement to the next point, the axis is directed vertically upward, $O^{(k)}Y^{(k)}$and $O^{(k)}Y^{(k)}$the axis forms $O^{(k)}Z^{(k)}$a right-handed coordinate system with the axes $O^{(k)}X^{(k)}$.

Designations:

3. $\underline{V}$ – the UAV velocity vector, and for the simplicity of the mathematical model during programming we will assume that $|\underline{V}| = const$;

4. $\nu^{(k)}$– y – the orientation angle of the UAV velocity vector at the end of $k$the -th guidance section;

5. $\underline{D}^{(k)}$– the beginning of the given inertial coordinate system on $k$the -th guidance section

$O^{(k)}$– is the origin of the given inertial coordinate system on k the guidance section.

Let us consider the movement of a UAV in a horizontal plane relative to a given inertial coordinate system in k-th guidance section, which is described by a system of linear differential equations

$$\{\dot{X}^{(k)} = X_x^{(k)}, \quad X^{(k)}(0) = X_0^{(k)}, \dot{Z}^{(k)} = V_z^{(k)}, \quad Z^{(k)}(0) = Z_0^{(k)}, \dot{V}_x^{(k)} = a_x^{(k)} \quad (8)$$

$$V^{(k)}(0) = V_{x0}^{(k)}, \dot{V}_z^{(k)} = a_z^{(k)}, \quad V_z^{(k)}(0) = V_{z0}^{(k)}, \quad (9)$$

where $\boldsymbol{X}^{(\boldsymbol{k})}, \boldsymbol{Z}^{(\boldsymbol{k})}$are the coordinates of the UAV in $\boldsymbol{k}$the -th coordinate system; $\boldsymbol{V}_{\boldsymbol{x}}^{(\boldsymbol{k})}, \boldsymbol{V}_{\boldsymbol{z}}^{(\boldsymbol{k})}$are the projections of the UAV velocity vector $\underline{\boldsymbol{V}}$onto the axes of the -th $\boldsymbol{k}$ coordinate system; $\boldsymbol{a}_{\boldsymbol{x}}^{(\boldsymbol{k})}, \boldsymbol{a}_{\boldsymbol{z}}^{(\boldsymbol{k})}$are the accelerations of the UAV in $\boldsymbol{k}$the -th coordinate system.

As an example, we present graphs of changes $\left(\boldsymbol{x}(\boldsymbol{t}), \boldsymbol{z}(\boldsymbol{t})\right)$ Figure 2.

In this case, time is considered from$\boldsymbol{t} \in [\mathbf{0,2}; \mathbf{4,4}]$ with step $\boldsymbol{h} = \mathbf{0,2}$. We will also provide

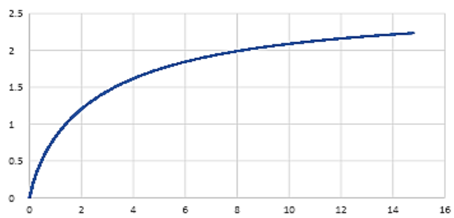

Figure 2 – Movement of the UAV at a constant speed and a changing angle with a certain step

Then the UAV movement graphs in different projections will be as follows Figure 3 $(\boldsymbol{x}(\boldsymbol{t});\ \boldsymbol{z}(\boldsymbol{t}))$:

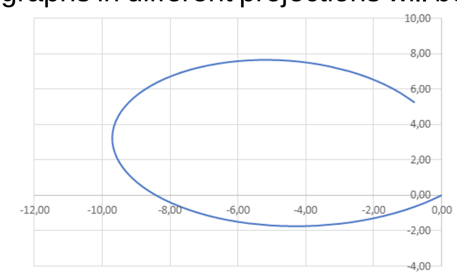

Figure 3 – UAV movement at constant speed and angular acceleration on a plane $(\boldsymbol{OZ})$

The examples cover two illustrative cases. In the first case, the UAV maintains a constant speed and heading angle; under these parameters, the trajectory forms a straight line, as predicted by the model. In the second case, the UAV flies at a constant speed but with constant angular acceleration, which leads to more complex curved trajectories when viewed from different planar projections. Both scenarios confirm that the mathematical structure and polynomial representation accurately reflect the underlying kinematic behavior.

The inertial coordinate system provides a rigorous and physically consistent framework for solving the UAV trajectory control problem, enabling an analytical description of motion in space and time while preserving the principle of inertia. Representing each flight segment in a local inertial frame and approximating the trajectory with fifth-degree polynomials allows precise satisfaction of boundary conditions on position, velocity, and acceleration. This ensures smooth segment-to-segment transitions and accurate passage through predefined waypoints.

Conclusion

The study presents a segmented approach to UAV trajectory determination, where each segment is defined by a local inertial coordinate system and a fifth-degree polynomial approximation. This structure ensures that the aircraft closely approaches the specified waypoints, minimizing the integral losses caused by control maneuvers and overload changes. The proposed method applies an analytical design of the optimal controller, which helps to reduce the computational complexity and supports efficient real-time computation.

The paper applies methods of analytical synthesis of tasks and optimization of UAV control systems for flight along successive sections, while it is necessary to take into account the requirements for satisfying real UAV flight conditions. The mathematical model of the UAV flight trajectory in one of the papers is based on the approximation of the UAV flight trajectory relative to the inertial system using a polynomial. In this case, boundary and initial conditions are specified to determine the coefficients of the polynomial.

Then the problem of determining the kinematic characteristics of the UAV's motion is considered, for this it is necessary to find the derivatives of the polynomial function. The first derivative with respect to time will give an expression for the speed, and the second - for the acceleration.

Thus, the fifth-degree polynomial model provides an effective representation of the UAV flight trajectory, allowing analytical determination of the position, speed and acceleration of the device at any time along the trajectory section, which is the basis for constructing an optimal control system.

The problem is then generalized using an inertial system, which provides lines describing the position of the UAV in space and time in an inertial frame of reference, and preserves the principle of inertia, i.e. if no external forces act on the UAV, it will move at a constant speed or remain at rest.

In practice, paired orthogonal lines are considered as the axes of the inertial system; for example, in three-dimensional space, axes designated as X, Y, Z, ko are used , which determine the position of the UAV in space and describe the movement of the UAV.

References

1. Alqudsi Y. UAV swarms: research, challenges, and future directions / Y. Alqudsi, M. Makaraci // Journal of Engineering and Applied Science. – 2025. – Vol. 72. – P. 12. https://doi.org/10.1186/s44147-025-00582-3.
2. UAV Path planning trends from 2000 to 2024: A bibliometric analysis and visualization / Q. Wu et al / Drones. – 2025. – Vol. 9. – P.128. https://doi.org/10.3390/drones9020128.
3. Online decision making and path planning framework for safe operation of unmanned aerial vehicles in urban scenarios / M. Quinones-Grueiro et al // International Journal of Prognostics and Health Management. – 2021. – Vol. 12. – P. 3. https://doi.org/10.36001/ijphm.2021.v12i3.2953.
4. A spherical vector-based adaptive evolutionary particle swarm optimization for UAV path planning under threat conditions / Y. Liu et al // Scientific Reports. – 2025. – Vol. 15. – P. 2116. https://doi.org/10.1038/s41598-025-85912-4.
5. Performance analysis of hybrid optimization approach for UAV path planning control using FOPID-TID controller and HAOAROA algorithm / N. Basil et al // Scientific Reports. – 2025. – Vol. 15. – P. 4840. https://doi.org/10.1038/s41598-025-86803-4.
6. Rgg-pso+: Random geometric graphs based particle swarm optimization method for UAV path planning / Y. Liu et al // International Journal of Computational Intelligence Systems. – 2024. – Vol. 17. – P. 127. https://doi.org/10.1007/s44196-024-00511-x.

7. Zhao Y. Survey on computational-intelligence-based UAV path planning / Y. Zhao, Z. Zheng, Y. Liu // Knowledge-Based Systems. – 2018. – Vol. 158. – P. 54-64. https://doi.org/10.1016/j.knosys.2018.05.033.
8. Aljalaud F. Optimizing Autonomous Multi-UAV Path Planning for Inspection Missions: A Comparative Study of Genetic and Stochastic Hill Climbing Algorithms / F. Aljalaud, Y. Alohali // Energies. – 2025. – Vol.18(1). – P. 50. https://doi.org/10.3390/en18010050.
9. Enhanced Multi-UAV Path Planning in Complex Environments With Voronoi-Based Obstacle Modelling and Q-Learning / W. Su et al // International Journal of Aerospace Engineering. – 2024. – Vol. 1. – P. 5114696. https://doi.org/10.1155/2024/5114696.
10. Hybrid PSO-HSA and PSO-GA algorithm for 3D path planning in autonomous UAVs / B. Abhishek et al // SN Applied Sciences. – 2020. – Vol. 2. – P. 1-16. https://doi.org/10.1007/s42452-020-03498-0.
11. A Comprehensive Review of Energy-Efficient Techniques for UAV-Assisted Industrial Wireless Networks / Yijia Zhang et al // Energies. – 2024. – Vol. 17(18). – P. 4737. https://doi.org/10.3390/en17184737.
12. Pounds P. Modelling and control of a large quadrotor robot / P. Pounds, R. Mahony, P. Corke // Control Engineering Practice. – 2010. – Vol. 18. – P. 691-699. https://doi.org/10.1016/j.conengprac.2010.02.008.

Acknowledgments

This research was supported by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan, grant number BR 249015/0224.

З.А. Кутпанова[1], Д.О. Кожахметова[2*], К.М. Сулейменов[3], Г.Н. Байсетов[4], Д.Н. Шандронов[5]

[1]Astana IT University,
010000, Қазақстан, Астана қ., проспект Мәңгілік Ел, 55/11
[2]Шәкәрім университеті,
071412, Қазақстан, Семей қ., Глинка к-сі, 20 А
[3]Л.Н. Гумилев атындағы Еуразия ұлттық университеті,
10000, Қазақстан, Астана қ., Сәтбаев к-сі, 2,
[4]Қазақстан инжиниринг R&D орталығы,
010000, Қазақстан, Астана қ., Әуезов көшесі, 2
[5]Қазақстан Республикасы Қорғаныс Министрлігі
Радиоэлектроника және байланыс әскери-инженерлік институты,
050040, Қазақстан Республикасы, Алматы қ.,Жандосов көшесі, 53
*e-mail: d.kojahmetova@shakarim.kz

ҰШҚЫШСЫЗ ҰШУ АППАРАТТАРЫНЫҢ ҰШУ ТРАЕКТОРИЯЛАРЫН СОҢҒЫ ШАРТТАРДЫ ЕСКЕРЕ ОТЫРЫП ОҢТАЙЛАНДЫРУ

Мақалада соңғы шарттары алдын ала берілген ұшқышсыз ұшу аппараттарының (ҰҰА) ұшу траекториясын оңтайландырудың өзекті мәселесі қарастырылады. Бесінші дәрежелі полиноммен жуықтау әдісін қолдана отырып, бірізді учаскелер бойымен ҰҰА-ның оңтайлы траекториясын құру тәсілі ұсынылған. Траекторияның әрбір учаскесінде жергілікті инерциялық координаталар жүйесін қолдану ерекшеліктерін ескеретін математикалық модель әзірленген. Қозғалысты оңтайландыру әдісі ұсынылады, ол берілген нүктелерге жету дәлдігін де, басқаруға кететін энергия шығындарын да сипаттайтын квадраттық функционалды минимизациялауға негізделген. Оңтайландыру есебінің шешімі оңтайлы контроллерді аналитикалық жобалау арқылы алынған. Энергия шығындары минималды болған жағдайда қажетті позициялау дәлдігін қамтамасыз ететін ҰҰА-ның оңтайлы бүйірлік үдеуін анықтауға арналған аналитикалық өрнектер келтірілген. Қозғалыстың үздіксіздігін қамтамасыз ету үшін траектория бөлімдері арасындағы координаталарды түрлендіру ерекшеліктері қарастырылған. Бастапқы шарттары берілген жағдайда ҰҰА-ның оңтайлы траекториясын есептеудің практикалық мысалы ұсынылған.

Түйін сөздер: *ұшқышсыз ұшу аппараттары; траекторияларды оңтайландыру; полиномдық жуықтау; инерциялық координаталар жүйелері; квадраттық функционалдар; оңтайлы басқару; таңдалған траектория.*

З.А. Кутпанова[1], Д.О Кожахметова[2]*, К.М. Сулейменов[3], Г.Н.Байсетов[4], Д.Н. Шандронов[5]
[1]Astana IT University, 010000, Казахстан, г. Астана, проспект Мәңгілік Ел, 55/11
[2]Шәкәрім университеті,
071412, Казахстан, г. Семей, Глинки, 20 А
[3]Евразийский Национальный университет им. Л.Н. Гумилева,
10000, Казахстан, г. Астана, Сатбаева 2
[4]Қазақстан инжиниринг R&D центр,
010000, Казахстан, г. Астана, Ауезова, 2
[5]Военно-инженерный институт радиоэлектроники и связи,
050040, Казахстан, г. Алматы, Жандосова, 53
*e-mail: d.kojahmetova@shakarim.kz

ОПТИМИЗАЦИЯ ТРАЕКТОРИЙ ПОЛЕТА БЕСПИЛОТНЫХ ЛЕТАТЕЛЬНЫХ АППАРАТОВ С УЧЕТОМ КОНЕЧНЫХ УСЛОВИЙ

Статья рассматривает актуальную проблему оптимизации траектории полёта беспилотных летательных аппаратов (БПЛА) с определенными конечными условиями. Представлен подход к построению оптимальной траектории БПЛА вдоль последовательных участков с использованием приближения по полиномиалу пятой степени. Разработана математическая модель, учитывающая особенности использования локальных инерционных координатных систем на каждом участке траектории. Предложен метод оптимизации движения, основанный на минимизации квадратической функциональности, характеризующей как точность достижения определенных точек, так и затраты энергии для управления. Решение проблемы оптимизации получено с использованием аналитического дизайна оптимального контроллера. Приведены аналитические выражения для определения оптимального бокового ускорения БПЛА, обеспечивающего требуемую точность позиционирования при минимальных затратах энергии. Рассмотрены особенности преобразования координат между секциями траектории для обеспечения непрерывности движения. Приведен практический пример расчета оптимальной траектории БПЛА при данных начальных условиях.

Ключевые слова: *беспилотные летательные аппараты; оптимизация траекторий; полиномиальное приближение; инерциальные системы координат; квадратично-функциональные; оптимальное управление; выбранная траектория.*

Авторлар туралы мәліметтер

Зарина Алмабековна Кутпанова – магистр, компьютерлік инженерия кафедрасының оқытушысы, Astana IT University, Астана, Қазақстан; e-mail: zarina.almabekkyzy@gmail.com. ORCID: https://orcid.org/0009-0007-4860-9244.

Динара Ошановна Кожахметова* – PhD, «Автоматтандыру және ақпараттық технологиялар» кафедрасының қауымдастырылған профессоры, Шәкәрім университеті, Семей, Қазақстан; e-mail: d.kojahmetova@shakarim.kz. ORCID: https://orcid.org/0000-0002-4327-3899.

Кенесары Машимович Сулейменов – PhD, физика-математика ғылымдарының кандидаты «Математика және компьютерлік модельдеу» кафедрасы Л.Н. Гумилев атындағы Еуразия Ұлттық Университеті, Астана, Қазақстан; e-mail: kenessarymath@gmail.com.

Гани Нуралиевич Байсеитов – техника ғылымдарының кандидаты, профессор «Казахстан инжиниринг» R&D орталығы, Астана, Қазақстан; e-mail: nesipova@rdke. https://orcid.org/0009-0001-0154-0071.

Дмитрий Шандронов – магистр, Қазақстан Республикасы Қорғаныс Министрлігі Радиоэлектроника және байланыс әскери-инженерлік институты, Алматы, Қазақстан; e-mail: shan_dima@mail.ru. ORCID: https://orcid.org/0009-0000-2361-2542.

Сведения об авторе

Зарина Алмабековна Кутпанова – магистр, преподаватель кафедры компьютерное инженерия; Astana IT University, Астана Казахстан; e-mail: zarina.almabekkyzy@gmail.com. ORCID: https://orcid.org/0009-0007-4860-9244.

Динара Ошановна Кожахметова* – PhD, ассоциированный профессор кафедры «Автоматизация и информационные технологии», университет Шәкәрім, Семей, Казахстан; e-mail: d.kojahmetova@shakarim.kz. ORCID: https://orcid.org/0000-0002-4327-3899.

Кенесары Машимович Сулейменов – PhD, кандидат физико-математических наук кафедры «Математическое и компьютерное моделирование», Евразийский Национальный университет им. Л.Н. Гумилева, Астана, Казахстан; e-mail:kenessarymath@gmail.com.

Гани Нуралиевич Байсеитов – кандидат технических наук, профессор R&D центр «Казахстан инжиниринг» Астана, Казахстан; e-mail:nesipova@rdke. ORCID: https://orcid.org/0009-0001-0154-0071.

Дмитрий Шандронов – магистр, Военно-инженерный институт радиоэлектроники и связи, Министерства Обороны Республики Казахстан, Алматы, Казахстан; e-mail: shan_dima@mail.ru. ORCID: https://orcid.org/0009-0000-2361-2542.

Information about the authors

Zarina Kutpanova – master of the Department «Computer engineering», Astana IT University, Astana, Kazakhstan; e-mail: zarina.almabekkyzy@gmail.com. ORCID: https://orcid.org/0009-0007-4860-9244.

Dinara Kozhakhmetova* – PhD, Associate Professor of the Department «Automation and Information Technologies», Shakarim University, Semey, Kazakhstan; e-mail: d.kojahmetova@shakarim.kz. ORCID: https://orcid.org/0000-0002-4327-3899.

Kenesary Suleymenov – PhD, Candidate of physical-mathematical sciences of the department «Mathematical and computer modelling». LN Gumilyov Eurasian National University, Astana, Kazakhstan; e-mail:kenessarymath@gmail.com.

Gani Baiseitov – candidate of technical Sciences, Professor R&D Center «Kazakhstan Engineering», Astana, Kazakhstan; e-mail:nesipova@rdke. ORCID: https://orcid.org/0009-0001-0154-0071.

Dmitry Shandronov – master of the Military Engineering Institute of Radio Electronics and Communications, Almaty, Kazakhstan; e-mail: shan_dima@mail.ru. ORCID: https://orcid.org/0009-0000-2361-2542.

Received 05.01.2026
Revised 19.02.2026
Accepted 20.02.2026

https://doi.org/10.53360/2788-7995-2026-1(21)-14

МРНТИ: 50.13.13

М.Ж. Ергеш[1]*, Б.Ж. Ергеш[1], А.Р. Оразаева[2], А. Талгат[2]
[1]Евразийский национальный университет им. Л.Н. Гумилёва,
010000, Казахстан, Астана, ул. Сатпаева, 2
[2]АО «Казахский университет технологии и бизнеса им. К. Кулажанова»,
010000, Казахстан, Астана, ул. Мухамедханова, 37А
*e-mail: shymkent90@mail.ru

ПОСТРОЕНИЕ И ЛИНГВИСТИЧЕСКИЙ АНАЛИЗ ВОПРОСНО-ОТВЕТНОГО КОРПУСА KAZAKH HISTORYQA

Аннотация: *Цифровизация среднего образования в Казахстане усиливает потребность в специализированных казахскоязычных ресурсах, поддерживающих интеллектуальные вопросно-ответные системы по школьным предметам. Однако существующие QA-датасеты в основном ориентированы на высокоресурсные языки и не охватывают содержание учебных дисциплин, в частности истории Казахстана. В данной работе представлен и подробно проанализирован корпус Kazakh HistoryQA – первый специализированный вопросно-ответный ресурс на казахском языке, созданный на основе шести официальных учебников по истории Казахстана для 5-10 классов. Корпус включает 208 тематических секций, около 180 тысяч токенов и 661 QA-пару с привязкой к точным координатам ответного спана, что обеспечивает совместимость со стандартными пайплайнами обучения моделей машинного чтения, включая библиотеку HuggingFace.*

Проведён многоуровневый анализ корпуса: статистика длины контекстов, вопросов и ответов, распределение материалов по классам и учебникам, типологизация вопросов на фактологические, хронологические, процедурные, причинно-следственные и сравнительные. Дополнительно выполнено морфологическое профилирование текста, построен частотный словарь и выделены предметные термины. Для оценки надежности корпуса как бенчмарка реализованы базовые модели выбора предложений (случайный выбор, TF-IDF, BM25), среди которых BM25 показал лучшие результаты. Представленный корпус создаёт основу для последующих исследований и разработки гибридных онтологически обогащённых QA-систем в сфере образования.

Ключевые слова: *Казахстанская история, OCR, QA, онтология, образовательная NLP, RAG, тезаурус, онтология.*