**DEVELOPMENT OF TECHNOLOGY OF MEAT PRODUCTS USING VEGETABLE RAW MATERIALS**
N. Abilmazhinova, A. Tayeva, Sh. Abzhanova, B. Dzhetpisbayeva

*The article presents the results of a study on the use of lentil flour in recipes of semi-finished meat products in order to create a product for herodietic purposes. The choice of lentil flour is based as an ingredient of meat-growing semi-finished products. The maximum possible dose of vegetable raw materials for semi-finished products has been established, which allows increasing their nutritional value while maintaining their favorable consumer properties.*

***Key words:*** *lentil flour, Herodian dish, Meat semi-finished products in pieces, nutritional value, combined product, spinach.*

МРНТИ: 30.19.15, 30.19.17

**M. Bukenov[1], Ye. Mukhametov[1],Ye. Ospanov[2] , S. Suleimenova[1]**
[1]Eurasian National University named after L.N. Gumilyov, Astana
[2]University Shakarim, Semey

## NON-AXISYMMETRIC EQUATIONS OF SHELL OSCILLATIONS WITH ATTACHED MASSES

***Abstract:*** *Thin-walled shell constructions that are capable of carrying intensive external loads are widely and variously used in aircraft engineering, rocket engineering, mechanical engineering, shipbuilding, construction and other sectors of the national economy. Modern requirements to reduction of weight-size parameters of flying and transport vehicles, industrial and civil constructions under condition of ensuring necessary durability to reliability have made calculation of their stress-strain state one of the actual problems of deformable solid body mechanics. Recently, a steady tendency has been observed to conduct complex theoretical and experimental studies of non-stationary deformation of real shell structures, usually consisting of several sections and characterized by the attached cargo, various supporting elements, holes and other complicating factors. The need to adequately define the dynamic behavior of shell systems with complex geometric and rheological structures leads to mathematical models that are beyond the traditional calculation schemes. Thus, for example, in [1] the effect of impulse waves on a cylindrical shell with solid bodies of different masses and moments of inertia at their ends is considered. The numerical analysis of dynamic reaction of objects is carried out within the framework of nonlinear theory of shells by V.V.Novozhilov. The vibration state and amplitude-frequency characteristics of a combined shell-and-rod structure with attached masses were numerically studied in [2, 3].*

*This paper presents a complete system of shell equations based on hypotheses of S.P.Timoshenko [4, 5]. The use of the modified theory of shell dynamics, which takes into account the inertia of rotation and transverse shift of a normal element, is due to the fact that polymer and composite materials, widely used in modern technology, are characterized by weak resistance to shear deformation, which are not taken into consideration by the classical theory of shells, and within the framework of this approach take nonzero values.*

***Key words:*** *two-dimensional thermoviscoelastic waves, stability of a difference scheme, convergence of a solution of a difference problem, indenter, deformation, stress tensor.*

### Introduction

Let us consider a thin shell, the middle surface of which is formed by rotation of a smooth curve $R(s)$ around the axis $Oz$ (Fig. 1 ). The radius-vector $\vec{r}(s,\varphi)$of an arbitrary point on the median surface is set as follows

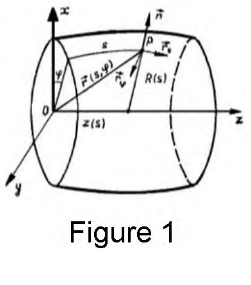


Figure 1

$$\vec{r}(s,\varphi) = (R(s)\cos\varphi, R(s), z(s)). \tag{1}$$

Directing vectors of the orthogonal local coordinate system in the point are entered as follows:

$$\vec{r}_s = \frac{\partial \vec{r}}{\partial s} = (R'\cos\varphi, R'\sin\varphi, z');$$
$$\vec{r}_\varphi = \frac{\partial \vec{r}}{\partial \varphi} = (-R\cos\varphi, R\sin\varphi, 0); \tag{2}$$
$$R' = \frac{dR(s)}{ds}; \quad z' = \frac{dz(s)}{ds};$$
$$R = R(s).$$

Using these vectors, we define the components of the metric tensor

$$g_{\alpha\beta}(\alpha, \beta = r, \varphi): \; g_{ss} = \vec{r}_s \cdot \vec{r}_s = (R')^2 + (z')^2 = 1;$$
$$g_{\varphi\varphi} = R^2; \; g_{\varphi s} = g_{s\varphi} = 0; \; g = |g_{\alpha\beta}| = R^2, \tag{3}$$

as well as Lamé coefficients $H_\alpha = \frac{1}{\sqrt{g_{\alpha\alpha}}} : H_s = 1, \; H_\varphi = \frac{1}{R}$.

The normal vector $\vec{n}$ will be set as

$$\vec{n} = \frac{1}{\sqrt{g}} \vec{r}_\varphi \times \vec{r}_s = (z'\cos\varphi, z'\sin\varphi, -R');$$
$$\vec{n}_s = \frac{\partial \vec{n}}{\partial s} = (z''\cos\varphi, z''\sin\varphi, -R'');$$
$$\vec{n}_\varphi = \frac{\partial \vec{n}}{\partial \varphi} = (-z'\sin\varphi, z'\cos\varphi, 0); \tag{4}$$
$$R'' = \frac{d^2R(s)}{ds^2}; \; z'' = \frac{d^2z(s)}{ds^2};$$

and the components of the second metric tensor $b_{\alpha\beta}$ will be calculated as $b_{\alpha\beta} = b_{\beta\alpha} = -\vec{n}_\alpha \cdot \vec{r}_\beta$:

$$b_{ss} = -\vec{n}_s \cdot \vec{r}_s = -R'z'' + z'R''$$

or, considering the relations between

$$z' = \sqrt{1-(R')^2}\,; \; z'' = \frac{-R'R''}{\sqrt{1-(R')^2}},$$

get

$$b_{ss} = \frac{R''}{\sqrt{1-(R')^2}} = \frac{1}{R_s^0};$$
$$b_{\varphi\varphi} = -\vec{n}_\varphi \cdot \vec{r}_\varphi = -Rz' = \frac{R^2}{R_\varphi^0}; \tag{5}$$
$$b_{s\varphi} = b_{\varphi s} = 0.$$

Here $R_s^0 = \frac{\sqrt{1-(R')^2}}{R''}$, $R_\varphi^0 = \frac{-R}{\sqrt{1-(R')^2}}$ - radii of curvature in directions $s$ and $\varphi$ respectively.

Mixed components of the second metric tensor are calculated as follows:

$$b_s^s = \frac{1}{R_s^0}; \; b_\varphi^\varphi = \frac{1}{R_\varphi^0}; \; b_\varphi^s = b_s^\varphi = 0\,. \tag{6}$$

Christophele's characters in the selected coordinate system are as follows

$$\Gamma_{ss}^s = \Gamma_{\varphi\varphi}^\varphi = \Gamma_{ss}^\varphi = \Gamma_{s\varphi}^s = \Gamma_{\varphi s}^s = 0; \; \Gamma_{\varphi\varphi}^s = -RR'; \; \Gamma_{\varphi s}^\varphi = \Gamma_{s\varphi}^\varphi = \frac{R'}{R}. \tag{7}$$

Now let us proceed to the finding of the basic ratios of the shell dynamics. The determining equations in tensor form for the shell of arbitrary curvature are given in [6]. Let us write the equations of motion, Hooke's law and Cauchy's relations, keeping only their linear part.

Motion equation:

$$\rho h \ddot{\vartheta}^\beta = \nabla_\alpha N^{\alpha\beta} - b_\alpha^\beta Q^\alpha + p^\beta;$$

$$\rho h\ddot{w}^{\beta} = \nabla_{\alpha} Q^{\alpha} + b_{\alpha\beta} N^{\alpha\beta} + q; \qquad (8)$$

$$\rho h\ddot{w}^{\beta} = \nabla_{\alpha} Q^{\alpha} + b_{\alpha\beta} N^{\alpha\beta} + q;$$

Here, the second derivative in time $t$ is marked by two dots above the letters. For recording the expressions in the right part (8), the rule of summing up by two repetitive indexes is used.

Symbols $\alpha, \beta$ denote variables $s, \varphi, \vartheta^{\beta}, w, \psi^{\beta}$ – the counter-variant components of the displacement vector and the normal angle of rotation, respectively. $N^{\alpha\beta}, Q^{\alpha}, M^{\alpha\beta}$ – counter-variant components of force and moment tensors; $p^{\beta}, q, m^{\beta}$ – intensities of forces (tangential and transverse), as well as moments distributed on the shell surface. $\nabla_{\alpha}$ – covariant derivative, $\rho$ – density of material, $h$ – thickness of the shell, $I = \dfrac{h^3}{12}$

Hook's Law:

$$N^{\alpha\beta} = h\mathrm{A}^{\alpha\beta\delta\mu} \varepsilon_{\delta\mu};$$
$$M^{\alpha\beta} = I\mathrm{A}^{\alpha\beta\delta\mu} \chi_{\delta\mu}; \qquad (9)$$
$$Q^{\alpha} = k^2 h C^{\alpha 3\delta 3} \left( w_{\beta} + \psi_{\beta} \right);$$
$$\mathrm{A}^{\alpha\beta\delta\mu} = \frac{C^{\alpha\beta\delta\mu} - C^{\alpha\beta 33} C^{33\delta\mu}}{C^{3333}};$$

The coefficients $C^{\alpha\beta\delta\mu}$ are Hook's elastic coefficient matrix:

$$\sigma^{ij} = C^{ijkl} \gamma_{kl} \quad \left( i, j, k, l = 1,2,3;\ s = 1,\ \varphi = 2 \right).$$

$k^2 = \dfrac{5}{6}$ – the shear coefficient in the S.P.Timoshenko theory. The tensors included in the right parts (9) are determined from Cauchy ratios:

$$\varepsilon_{\alpha\beta} = \frac{1}{2}\left( l_{\alpha\beta} + l_{\beta\alpha} \right);\ \ \chi_{\alpha\beta} = \frac{1}{2}\left( \chi'_{\alpha\beta} + \chi'_{\beta\alpha} \right);$$
$$\varepsilon_{r\alpha} = \frac{1}{2}\left( w_{\alpha} + \psi_{\alpha} \right);\ \ l_{\alpha\beta} = \nabla_{\alpha} \vartheta_{\beta} + b_{\alpha\beta} w; \qquad (10)$$
$$\chi'_{\alpha\beta} = \nabla_{\alpha} \psi_{\beta};\ \ w_{\alpha} = \nabla_{\alpha} w + b_{\alpha\beta} \vartheta^{\beta}.$$

The representation of relations (8) – (10) in terms of physical components of displacements, forces and moments is illustrated by the example of the first equation in (8) at $\beta = s$. By definition of the covariant derivative from the invariant tensor is obtained as follows

$$\nabla_{\alpha} N^{\alpha\beta} = \frac{\partial N^{\alpha\beta}}{\partial x^{\alpha}} + N^{l\beta} \Gamma^{\alpha}_{l\alpha} + N^{\alpha l} \Gamma^{\beta}_{l\alpha}$$

or at $\beta = s$

$$\nabla_{\alpha} N^{\alpha s} = \frac{\partial N^{ss}}{\partial s} + \frac{\partial N^{\varphi s}}{\partial \varphi} + N^{ss}\left( 2\Gamma^{s}_{ss} + \Gamma^{\varphi}_{s\varphi} \right) + N^{\varphi s}\left( 3\Gamma^{s}_{\varphi s} + \Gamma^{\varphi}_{\varphi\varphi} \right) + N^{\varphi\varphi} \Gamma^{s}_{\varphi\varphi} + N^{s\varphi} \Gamma^{s}_{s\varphi} =$$
$$= \frac{\partial N^{ss}}{\partial s} + \frac{\partial N^{\varphi s}}{\partial \varphi} + \frac{R'}{R} N^{ss} - R'RN^{\varphi\varphi}$$

Here we used formulas (7). Further from (6) we have

$$b^{s}_{\alpha} Q^{\alpha} = b^{s}_{s} Q_{s} + b^{s}_{\varphi} Q^{\varphi} = \frac{1}{R^{0}_{s}} Q^{s}.$$

Then the first equation in (8) takes the form

$$\rho h \ddot{\vartheta}^{s} = \frac{\partial N^{ss}}{\partial s} + \frac{\partial N^{\varphi s}}{\partial \varphi} + \frac{R'}{R} N^{ss} - RR'N^{\varphi\varphi} - \frac{1}{R^{0}_{s}} Q^{s} + p^{s}. \qquad (11a)$$

Similar calculations lead to the following system of equations:

$$\rho h\ddot{\vartheta}^{\varphi}=\frac{\partial N^{s\varphi}}{\partial s}+\frac{\partial N^{\varphi\varphi}}{\partial\varphi}+3\frac{R'}{R}N^{s\varphi}-\frac{1}{R_{\varphi}^{0}}Q^{\varphi}+p^{\varphi};$$

$$\rho h\ddot{w}=\frac{\partial Q^{s}}{\partial s}+\frac{\partial Q^{\varphi}}{\partial\varphi}+\frac{R'}{R}Q^{s}+\frac{1}{R_{s}^{0}}N^{ss}+\frac{R^{2}}{R_{s}^{0}}N^{\varphi\varphi}+q; \tag{11b}$$

$$\rho h\ddot{\psi}^{s}=\frac{\partial M^{ss}}{\partial s}+\frac{\partial M^{\varphi s}}{\partial\varphi}+\frac{R'}{R}M^{ss}-RR'M^{\varphi\varphi}-Q^{s}+m^{s};$$

$$\rho h\ddot{\psi}^{\varphi}=\frac{\partial M^{s\varphi}}{\partial s}+\frac{\partial M^{\varphi\varphi}}{\partial\varphi}+3\frac{R'}{R}M^{s\varphi}-Q^{\varphi}+m^{\varphi}.$$

Physical components of tensors are determined by the following formulas

$$a_{\alpha(\phi)}=a_{(\phi)}^{\alpha}=\frac{a^{\alpha}}{H_{\alpha}}=a_{\alpha}H_{\alpha};\ \mathrm{T}_{\alpha\beta(\phi)}=\mathrm{T}_{\alpha(\phi)}^{\beta}=\mathrm{T}_{(\phi)}^{\alpha\beta}=\mathrm{T}_{\alpha\beta}H_{\alpha}H_{\beta}=\frac{\mathrm{T}^{\alpha\beta}}{\left(H_{\alpha}H_{\beta}\right)}=\frac{\mathrm{T}_{\alpha}^{\beta}H_{\alpha}}{H_{\beta}}. \tag{12}$$

Substitute in (11) all components of the tensors with their physical equivalents (sign "$\varphi$" suppressed):

$$\rho h\ddot{\vartheta}_{s}=\frac{\partial N_{ss}}{\partial s}+\frac{1}{R}\frac{\partial N_{\varphi s}}{\partial\varphi}+\frac{R'}{R}N_{ss}-\frac{R'}{R}N_{\varphi\varphi}-\frac{1}{R_{s}^{0}}Q_{s}+p_{s};$$

$$\frac{\rho h\ddot{\vartheta}_{\varphi}}{R}=\frac{1}{R}\frac{\partial N_{s\varphi}}{\partial s}+\frac{1}{R^{2}}\frac{\partial N_{\varphi\varphi}}{\partial\varphi}+3\frac{R'}{R^{2}}N_{s\varphi}-\frac{1}{RR_{\varphi}^{0}}Q_{\varphi}+p_{\varphi};$$

$$\rho h\ddot{w}=\frac{\partial Q_{s}}{\partial s}+\frac{1}{R}\frac{\partial Q_{\varphi}}{\partial\varphi}+\frac{R'}{R}Q_{s}+\frac{1}{R_{s}^{0}}N_{ss}+\frac{1}{R_{\varphi}^{0}}N_{\varphi\varphi}+q; \tag{13}$$

$$\rho I\ddot{\psi}_{s}=\frac{\partial M_{ss}}{\partial s}+\frac{1}{R}\frac{\partial M_{\varphi s}}{\partial\varphi}+\frac{R'}{R}M_{ss}-\frac{R'}{R}M_{\varphi\varphi}-Q_{s}+m_{s};$$

$$\frac{\rho I\ddot{\psi}_{\varphi}}{R}=\frac{1}{R}\frac{\partial M_{s\varphi}}{\partial s}+\frac{1}{R^{2}}\frac{\partial M_{\varphi\varphi}}{\partial\varphi}+3\frac{R'}{R^{2}}M_{s\varphi}-\frac{1}{R}Q_{\varphi}+m_{\varphi}.$$

Finally, let us enter the variable $y=R\varphi$ and rewrite the system (13) in a divergent form:

$$\rho h\ddot{\vartheta}_{s}=\frac{1}{R}\frac{\partial\left(RN_{ss}\right)}{\partial s}+\frac{\partial N_{\varphi s}}{\partial y}-\frac{R'}{R}N_{\varphi\varphi}-\frac{1}{R_{s}^{0}}Q_{s}+p_{s};$$

$$\rho h\ddot{\vartheta}_{\varphi}=\frac{1}{R}\frac{\partial\left(RN_{s\varphi}\right)}{\partial s}+\frac{\partial N_{\varphi\varphi}}{\partial y}+2\frac{R'}{R}N_{s\varphi}-\frac{1}{R_{\varphi}^{0}}Q_{\varphi}+p_{\varphi};$$

$$\rho h\ddot{w}=\frac{1}{R}\frac{\partial\left(RQ_{s}\right)}{\partial s}+\frac{\partial Q_{\varphi}}{\partial y}+\frac{1}{R_{s}^{0}}N_{ss}+\frac{1}{R_{\varphi}^{0}}N_{\varphi\varphi}+q; \tag{14}$$

$$\rho I\ddot{\psi}_{s}=\frac{1}{R}\frac{\partial\left(RM_{ss}\right)}{\partial s}+\frac{\partial M_{\varphi s}}{\partial y}-\frac{R'}{R}M_{\varphi\varphi}-Q_{s}+m_{s};$$

$$\rho I\ddot{\psi}_{\varphi}=\frac{1}{R}\frac{\partial\left(RM_{s\varphi}\right)}{\partial s}+\frac{\partial M_{\varphi\varphi}}{\partial y}+2\frac{R'}{R}M_{s\varphi}-Q_{\varphi}+m_{\varphi}.$$

Now let's consider ratios (10). By definition of a derivative

$$\nabla_{\alpha}u_{\beta}=\frac{\partial u_{\beta}}{\partial x^{\alpha}}-\Gamma_{\beta\alpha}^{\lambda}u_{\lambda};\ \nabla_{\alpha}w=\frac{\partial w}{\partial x^{\alpha}}.$$

Then, using expressions (7), from (10), we have

$$\varepsilon_{ss}=\frac{\partial\vartheta_{s}}{\partial s}-\frac{w}{R_{s}^{0}};\ \varepsilon_{s\varphi}=\frac{1}{2}\left(\frac{\partial\vartheta_{s}}{\partial y}+\frac{\partial\vartheta_{\varphi}}{\partial s}\right)-\frac{R'\vartheta_{\varphi}}{R};$$

$$\varepsilon_{\varphi\varphi}=\frac{\partial\vartheta_{\varphi}}{\partial y}+\frac{R'}{R}\vartheta_{s}-\frac{w}{R_{\varphi}^{0}};\ \varepsilon_{rs}=\frac{1}{2}\left(\frac{\partial w}{\partial s}+\frac{\vartheta_{s}}{R_{s}^{0}}+\psi_{s}\right);$$

$$\varepsilon_{r\varphi} = \frac{1}{2}\left(\frac{\partial w}{\partial y} + \frac{\vartheta_\varphi}{R_\varphi^0} + \psi_s\right); \;\; \chi_{ss} = \frac{\partial \psi_s}{\partial s};$$

$$\chi_{\varphi\varphi} = \frac{\partial \psi_\varphi}{\partial y} + \frac{R'}{R}\psi_s; \;\; \chi_{s\varphi} = \frac{1}{2}\left(\frac{\partial \psi_s}{\partial y} + \frac{\partial \psi_\varphi}{\partial s}\right) - \frac{R'}{R}\psi_\varphi.$$

Turning on the formulas (12) to the physical components, we get

$$\varepsilon_{ss} = \frac{\partial \vartheta_s}{\partial s} - \frac{1}{R_s^0}w; \;\; \varepsilon_{s\varphi} = \frac{1}{2}\left(\frac{\partial \vartheta_s}{\partial y} + \frac{\partial \vartheta_\varphi}{\partial s}\right) - \frac{R'}{R}\vartheta_\varphi;$$

$$\varepsilon_{\varphi\varphi} = \frac{\partial \vartheta_\varphi}{\partial y} + \frac{R'}{R}\vartheta_s - \frac{1}{R_\varphi^0}w; \;\; \varepsilon_{rs} = \frac{1}{2}\left(\frac{\partial w}{\partial s} + \frac{\vartheta_s}{R_s^0} + \psi_s\right); \tag{15}$$

$$\varepsilon_{r\varphi} = \frac{1}{2}\frac{\partial w}{\partial y} + \frac{\vartheta_\varphi}{R_\varphi^0} + \psi_\varphi; \;\; \chi_{ss} = \frac{\partial \psi_s}{\partial s}; \;\; \chi_{\varphi\varphi} = \frac{\partial \psi_\varphi}{\partial y} + \frac{R'}{R}\psi_s;$$

$$\chi_{s\varphi} = \frac{1}{2}\left(\frac{\partial \psi_\varphi}{\partial s} + \frac{\partial \psi_s}{\partial y}\right) - \frac{R'}{R}\psi_\varphi;$$

In the future we will consider orthotropic material, so Hooke's law for the physical components of force, moment, and deformation tensors is as follows [7]:

$$N_{ss} = \frac{\mathrm{E}_1 h}{1-\nu_1\nu_2}\left(\varepsilon_{ss} + \nu_2\varepsilon_{\varphi\varphi}\right); \;\; N_{\varphi\varphi} = \frac{\mathrm{E}_2 h}{1-\nu_1\nu_2}\left(\nu_1\varepsilon_{ss} + \varepsilon_{\varphi\varphi}\right);$$

$$N_{s\varphi} = N_{\varphi s} = 2hG_{s\varphi}\varepsilon_{s\varphi}; \; Q_s = 2k^2hG_{rs}\varepsilon_{rs}; \tag{16}$$

$$Q_\varphi = 2k^2hG_{r\varphi}\varepsilon_{r\varphi}; \;\; M_{ss} = \frac{\mathrm{E}_1 I}{1-\nu_1\nu_2}\left(\chi_{ss} + \nu_2\chi_{\varphi\varphi}\right);$$

$$M_{\varphi\varphi} = \frac{\mathrm{E}_2 I}{1-\nu_1\nu_2}\left(\nu_1\chi_{ss} + \chi_{\varphi\varphi}\right); M_{s\varphi} = M_{\varphi s} = 2IG_{s\varphi}\chi_{s\varphi}.$$

Here $\mathrm{E}_1, \mathrm{E}_2, \nu_1, \nu_2$ − Jungian modules and Poisson's coefficients in the directions $s$ and $\varphi$ respectively, and $\mathrm{E}_1\nu_2 = \mathrm{E}_2\nu_1$; $G_{s\varphi}, G_{rs}, G_{r\varphi}$ − shear moduli.

Thus equations (14) - (16) comprise the complete system for determining displacements, normal angles, forces, moments and deformations.

Let's consider the left end of the shell with an absolutely hard drive attached to it, the thickness $2H$, radius $R$ and mass $M$. Since the linear equations of the theory of shells are used, it is assumed that the mass makes small oscillations under the action of external forces and reaction of the shell. As the origin of the movable reference system related to mass, let us choose the center of inertia of the body $O$ and direct the axes $OA, OB, OC$ along the main axes of mass inertia (Fig. 2). Let us also introduce a stationary reference system $Ox'y'z'$, coinciding at the starting point of time with the system.

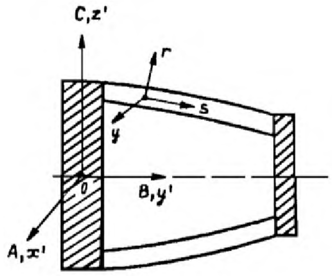


Figure 2

The law of mass motion in vector form has the form of [8]:

$$\dot{\vec{\mathrm{P}}} = \vec{F}_0 + \vec{F}; \;\; \dot{\vec{L}} = \vec{K}_0 + \vec{K}, \tag{17}$$

где $\vec{\mathrm{P}}$ − total body impulse; $\vec{L}$ − impulse moment; $\vec{F}_0$ и $\vec{K}_0$ − main vector and main force moment acting on the mass from the shell side; $\vec{F}$ и $\vec{K}$ − main vector and moment of external forces.

Equations (17) refer to a fixed coordinate system and derivatives $\dot{\vec{\mathrm{P}}}$ and $\dot{\vec{L}}$ represent a change in time of vectors $\vec{\mathrm{P}}$ and $\vec{L}$ in relation to this system. Meanwhile, the simplest relationship between the components of solid state rotational moment $\vec{L}$ and angular velocity components occurs in a moving coordinate system $OABC$. Therefore we transform the equations of motion to moving coordinates. For this purpose we apply the equation of transformation of the time derivative of an arbitrary vector $\vec{D}$ at transition from a stationary system to a rotating one:

$$\left(\frac{d\vec{D}}{dt}\right)_{пространство} = \left(\frac{d\vec{D}}{dt}\right)_{тело} + \left(\vec{\omega}\times\vec{D}\right). \tag{18}$$

where, $\vec{\omega}$ − body angular velocity vector.

Due to the small amplitudes of mass oscillations and the striking nature of the system's stimulation, it is possible to ignore the difference between the decomposition of any vector (and its derivative in time) on the axes of the moving and stationary coordinate system. Then the law of mass movement in vector form takes the following form

$$M\ddot{\vec{V}} = \vec{F}_0 + \vec{F};\; I_0\dot{\vec{\omega}} = \vec{K}_0 + \vec{K}, \tag{19}$$

где $\vec{V}$ − center radius vector; $I_0$ − body inertia tensor.

Since the system's axes $OABC$ are directed along the main axes of inertia, the inertia tensor $I_0$ has a diagonal view and its components can be easily calculated.

$$I_{01} = I_{03} = \left(K^2 + \frac{4H^2}{3}\right)\frac{M}{4},\; I_{02} = \frac{MR^2}{2}.$$

Now let's consider the ways of orientation of the moving trihedron $OABC$ relatively to fixed $Ox'y'z'$. There is a well known method of determining the orientation of the trihedron $OABC$ with respect to $Ox'y'z'$ using Euler angles $\theta,\varphi,\psi$ through $\theta$ and $\varphi$ the polar angles of the axis $OC$, and through $\psi$ -the angle between $z'OC$ plane and $COA$ plane. Let us indicate through $\omega_1,\omega_2,\omega_3$ the components of the angular velocity vector respectively in the axes $OA, OB, OC$:

$$\begin{aligned} \omega_1 &= \dot{\theta}\sin\psi - \dot{\varphi}\sin\theta\cos\varphi; \\ \omega_2 &= \dot{\theta}\cos\psi + \dot{\varphi}\sin\theta\sin\varphi; \\ \omega_3 &= \dot{\varphi}\cos\theta + \dot{\varphi}. \end{aligned} \tag{20}$$

If at some point in time the trihedron $OABC$ coincides with the trihedron $Ox'y'z'$ so that $\theta = \varphi = \psi = 0$, from (20) it follows that $\omega_1 = 0,\, \omega_2 = \dot{\theta},\, \omega_3 = \dot{\varphi} + \dot{\psi}$. The component turns out to be equal to zero, no matter what the values $\dot{\theta},\dot{\varphi},\dot{\psi}$, are, which in general is wrong. That is why it is inconvenient to use Euler's corners in those cases where the trihedron $OABC$ coincides with the trihedron $Ox'y'z'$ at some point in time, except for those cases when the vector $\vec{\omega}$ lies in a plane $Oy'z'$ at this moment.

Let's consider another way to determine the trihedron orientation [9, 10]. As it was stated at the beginning, the trihedron $OABC$ coincides with $Ox'y'z'$. Transition of the trihedron to the final position is carried out by performing the following three consecutive operations: turning it by an angle $\theta_1$ around the axis $OA \equiv Ox'$, then turning it by an angle $\theta_2$ around the axis $OB$ in the new position and, finally, by an angle $\theta_3$ around the axis $OC$ in the new position. Let's define through G the matrix of guiding cosines of axes $OA, OB, OC$: in relation to fixed axes $Ox',Oy',Oz'$. If $\vec{X} = (x_1,x_2,x_3)$- the coordinates of the vector in the system $OABC$, and $\vec{Y} = (y_1,y_2,y_3)$ - its coordinates in the system $Ox'y'z'$, then

$$\vec{X} = G\vec{Y}. \tag{21}$$

Let's result without the proof the following lemma: if the trihedron $OABC$ rotates on an angle $\theta$ near an axis $OA$, the matrix of guide cosines in new position $G_1$ is set by the formula

$$G_1 = BG,$$

where

$$B_1(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}. \tag{22}$$

Similarly, when turning by angle $\theta$ around an axis $OB$, we get

$$G_2 = B_2 G,$$

where

$$B_2(\theta) = \begin{pmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{pmatrix}, \tag{23}$$

and when rotates by angle $\theta$ around the axis $OC$.

$$G_3 = B_3 G,$$

where

$$B_2(\theta) = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{24}$$

$B_1, B_2, B_3$ - orthogonal matrices, besides, $B_i^{-1}(\theta) = B_i(-\theta),\ i = 1,2,3.$

If the starting position $OABC$ is the same $Ox'y'z'$ and the end position is reached by turning by angles $\theta_1, \theta_2, \theta_3$, , then

$$G = B_3(\theta_3) B_2(\theta_2) B_1(\theta_1), \tag{25}$$

or

$$G = \begin{pmatrix} c_2 c_3 & c_1 s_3 + s_1 s_2 c_3 & s_1 s_3 + c_1 s_2 c_3 \\ -c_2 c_3 & c_1 c_3 + s_1 s_2 s_3 & s_1 c_3 + c_1 s_2 s_3 \\ s_2 & -s_1 c_2 & c_1 c_2 \end{pmatrix}. \tag{26}$$

Here, $c_i$ and $s_i, i = 1,2,3,$ definition for $\cos\theta_i$ и $\sin\theta_i$.

The infinitesimal rotation associated with $\vec{\omega}$, should be considered as a set of three consecutive infinitesimal rotations with angular velocities $\dot{\theta}_1, \dot{\theta}_2\, \dot{\theta}_3$. Then, according to the known vector property of infinitely small rotations, we can consider $\vec{\omega}$ the sum of three separate angular velocity vectors $\vec{\dot{\theta}}_1 = (\dot{\theta}_1, 0, 0)$, $\vec{\dot{\theta}}_2 = (0, \dot{\theta}_2, 0)$, $\vec{\dot{\theta}}_3 = (0, 0, \dot{\theta}_3)$ recorded in different coordinate systems. However, the components of these vectors with respect to any coordinate system can be obtained using orthogonal transformations $B_1, B_2, B_3$. Let's write out components of a vector in the system connected with a moving body:

$$\vec{\omega} = B_3(\theta_3) B_2(\theta_2) B_1(\theta_1) \vec{\dot{\theta}}_1 + B_3(\theta_3) B_2(\theta_2) \vec{\dot{\theta}}_2 + B_3(\theta_3) \vec{\dot{\theta}}_3 \tag{27}$$

and detailed:

$$\begin{aligned} \omega_1 &= c_2 c_3 \dot{\theta}_1 + s_3 \dot{\theta}_2; \\ \omega_2 &= -c_2 s_3 \dot{\theta}_1 + c_3 \dot{\theta}_2; \\ \omega_3 &= s_2 \dot{\theta}_1 + \dot{\theta}_3. \end{aligned} \tag{28}$$

Let's resolve the system (28) regarding $\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3$:

$$\dot{\theta}_1 = \frac{(c_3\omega_1 - s_3\omega_2)}{c_2};$$
$$\dot{\theta}_2 = s_3\omega_1 + c_3\omega_2; \tag{29}$$
$$\dot{\theta}_3 = \frac{-s_2(c_3\omega_1 - s_3\omega_2)}{c_2} + \omega_3.$$

Therefore, there is a one-to-one correspondence between $\vec{\omega}$ and $\dot{\vec{\theta}}$ vectors for all $\theta_1, \theta_2, \theta_3$, except $\cos\theta_2 = 0$. Thus, in the case of small fluctuations of mass near the equilibrium position, this method of trihedron orientation $OABC$ relatively to $Ox'y'z'$ excludes those "undesirable paradoxes" that were encountered when using Euler angles.

Taking out the second order members in (26) and (28), we get

$$G = \begin{pmatrix} 1 & \theta_3 & -\theta_2 \\ -\theta_3 & 1 & \theta_1 \\ \theta_2 & -\theta_1 & 1 \end{pmatrix}, \quad \vec{\omega} = \dot{\vec{\theta}}. \tag{30}$$

Let us define $\Gamma$ as the contact surface of the shell with the mass, through $\vec{\sigma} = (\sigma_{s\varphi}, \sigma_{ss}, \sigma_{rs})$- the components of the stress tensor. Then, the reaction of the shell is given by the following formulas:

$$\vec{F}_0 = \iint_\Gamma B_2^{-1}(\varphi)\cdot B_1^{-1}(-\beta)\cdot\vec{\sigma}\cdot d\gamma; \tag{31}$$
$$\vec{K}_0 = \iint_\Gamma \left(\vec{r}_0 + B_2^{-1}(\varphi)\cdot B_1^{-1}(-\beta)\vec{\mathrm{E}}_3\right)\times\left(B_2^{-1}(\varphi)B_1^{-1}(-\beta)\vec{\sigma}\right)d\gamma,$$

where $\vec{r}_0 = \left(R\sin\varphi, \mathrm{H} - \frac{h}{2}\sin\beta, R\cos\varphi\right)^{\mathrm{T}}$, $\vec{\mathrm{E}}_3 = (0,0,1)^{\mathrm{T}}$ and $\beta$ -angle between the positive axes $z'$ and $r$ and the points of junction of the mass with the shell.

Through $\vec{N} = (N_{s\varphi}, N_{ss}, Q_s)$ and $\vec{M} = (M_{s\varphi}, M_{ss}, 0)$ signify the components of the force and moment tensors on $\Gamma$, taking into account the known expressions:

$$\vec{N} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \vec{\sigma} dr; \quad \vec{M} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \vec{\sigma}\, r dr; \tag{32}$$

and ratio $d\gamma = Rdrd\varphi$ of formula (31) transformed to:

$$\vec{F}_0 = \int_0^{2\pi} B_2^{-1}(\varphi)\cdot B_1^{-1}(-\beta)\cdot\vec{N}Rd\varphi;$$

$$\vec{K}_0 = \int_0^{2\pi}\left[\vec{r}_0\times\left(B_2^{-1}(\varphi)\cdot B_1^{-1}(-\beta)\vec{N}\right) + \left(B_2^{-1}(\varphi)B_1^{-1}(-\beta)\vec{\mathrm{E}}_3\right)\times\left(B_2^{-1}(\varphi)B_1^{-1}(-\beta)\vec{M}\right)\right]Rd\varphi. \tag{33}$$

The formula (19), (30), (33) fully describe the linearized law of solid mass movement.

Let us now consider the boundary condition at the left end of the shell. The displacement of the mass points corresponding to the points of the median shell surface is summed up by the displacement of the center of the mass and the displacement due to rotation. Therefore, the vector of displacement of these points in the local shell basis $y, s, r$ can be obtained by the formula:

$$\vec{U} = B_1(-\beta)B_2(\varphi)\left[\vec{V} + \left(G^{-1}\vec{r}_0 - \vec{r}_0\right)\right], \ \varphi\in[0, 2\pi],$$

where $\vec{U} = (\upsilon_\varphi, \upsilon_s, w)$.

Let us indicate $\vec{n}$ as a single vector of the external normal to the median surface, where $n_\varphi, n_s, n_r$- its components in the local basis. In non-deformable state $\vec{n}$ it coincides with $\vec{\mathrm{E}}_3$, and after deformation it is expressed by the following formula:

$$\vec{n} = B_1(-\beta)B_2(\varphi)G^{-1}B_2^{-1}(\varphi)B_1^{-1}(-\beta)\vec{\mathrm{E}}_3.$$

It's not hard to see that $\psi_\varphi = n_\varphi, \psi_s = n_s$.

Equations of motion of the mass attached to the right face of the shell are recorded in a similar way (see Fig. 2):

$$M'\ddot{\vec{V}}' = -\int_0^{2\pi} B_2^{-1}(\varphi)\cdot B_1^{-1}(-\beta')\cdot \vec{N}' R\,d\varphi + \vec{F}';$$

$$I_0'\ddot{\vec{\theta}}' = -\int_0^{2\pi} \left[\vec{r}_0' \times \left(B_2^{-1}(\varphi)\cdot B_1^{-1}(-\beta')\vec{N}'\right) + \left(B_2^{-1}(\varphi)B_1^{-1}(-\beta')\vec{\mathrm{E}}_3\right) \times \left(B_2^{-1}(\varphi)B_1^{-1}(-\beta')\vec{M}'\right)\right] R\,d\varphi + \vec{K}'.$$

Here, ´ indicates that the marked value refers to the second mass and

$$\vec{r}_0' = \left(R\sin\varphi, -\mathrm{H} + \frac{h}{2}\sin\beta, R\cos\varphi\right)^{\mathrm{T}}.$$

The conjugation conditions of the mass to the shell in this case are as follows:

$$\vec{U}' = B_1(-\beta')B_2(\varphi)\left[\vec{V}' + \left((G')^{-1}\vec{r}_0' - \vec{r}_0'\right)\right];$$

$$\vec{n}' = B_1(-\beta')B_2(\varphi)(G')^{-1}B_2^{-1}(\varphi)B_1^{-1}(-\beta')\vec{\mathrm{E}}_3,\ \varphi \in [0, 2\pi].$$

## References


1. Non-Stationary aeroelasticity of the thin-walled constructions (1981) − Karmishin, A.V.; Skurlatov, E.D.; Startsev, V.G. et al. Edited by Karmishin A.V. − M.: Mashinostroenie, p. 240.
2. Bukenov M., Azimova D. (2015). Estimates for maxwell viscoelastic medium "in tension-rates". Eurasian mathematical journal. – L.N. Gumilyov Eurasian National University Nur-Sultan, Kazakhstan. Volume 10, Number 2 (2019), p. 30 – 36. DOI: https://doi.org/10.32523/2077-9879-2019-10-2-30-36
3. Bukenov M.M., Adamov A.A., Mukhametov E.M. (2019). Two-dimensional thermovisco-elastic waves in layered media. Bulletin of the Karaganda University. Mathematics series, 2 (94), 106-114. https://doi.org/10.31489/2019M2/106-114
4. Bulgaru O.E., Rybakova G.A. (1983). Numerical analysis of the wave fields and amplitude-frequency characteristics of the composite cylindrical shells. − News of the Academy of Sciences of MSSR. Series of Physical and Technical Sciences, No 2, p.4 − 10. [in Russian].
5. Bukenov M., Ibrayev A., Zhussupova D., Azimova D. (2017). Numerical solution of a problem on bending oscillation of a rod. Bulletin of the Karaganda University. Mathematics series, № 2(86), 32-36.
6. Volmir A.S. (1976). Shells in a fluid and gas flow. Tasks of aeroelasticity. − Moscow: Nauka, p. 416. [in Russian].
7. Volmir, A.S. (1972). Nonlinear dynamics of the plates and shells. Moscow: Nauka, p. 432. [in Russian].
8. Ricarda R.B., Tetere G.A. (1974). Stability of the composite shells. − Riga: Zinatne, p. 310. [in Russian].
9. Landau L.D., Livshits E.M. (1965). Mechanics. − Moscow: Nauka, p. 206. [in Russian].
10. Pars Jl.A. (1971). Analytical dynamics. − Moscow: Nauka, p. 635. [in Russian].


## ҚОСЫЛҒАН МАССАЛАРЫ БАР СИММЕТРИЯЛЫ ЕМЕС ҚАБЫҚШАЛАРДЫҢ ТЕРБЕЛІСТЕРІНІҢ ТЕҢДЕУЛЕРІ


М.М. Букенов, Е.М. Мухаметов, Е.А. Оспанов, С.Т. Сулейменова



*Қарқынды сыртқы жүктемелердің әсеріне төтеп бере алатын жұқа қабырғалы қабық конструкциялары авиақұрылыста, машина жасау, кеме жасау, Құрылыс және халық шаруашылығының басқа да салаларында ракеталық техникада кең және әр түрлі қолдануды табады. Ұшу және көлік аппараттарының, өнеркәсіптік және азаматтық құрылыстардың салмақ габаритті көрсеткіштерін төмендетуге қойылатын заманауи талаптар сенімділікке қажетті беріктікті қамтамасыз ету жағдайында олардың кернеулі-деформацияланатын күйлерін қатты дене механикасының деформацияланатын өзекті мәселелерінің бірі ретінде есептеу жасады.*

*Соңғы уақытта, бірнеше секциялардан тұратын және жалғанған жүктің болуымен сипатталатын, әртүрлі нығайтушы элементтердің, тесіктерді әлсірететін және басқа да қиындататын факторлардың нақты қабық конструкцияларының стационарлық емес деформациялануына кешенді теориялық-эксперименталдық зерттеулер жүргізудің тұрақты үрдісі байқалды. Күрделі геометриялық және реологиялық құрылымдар туралы қабықтық жүйелердің динамикалық мінез-құлқын барабар сипаттау қажеттілігі дәстүрлі есептік схемалар шеңберінен шығатын математикалық модельдерге әкеледі. Мысалы, [1] шетінде әртүрлі массалары мен инерция моменттері бар қатты денелер бар цилиндрлік қабықтағы соққы толқындарының әрекеті қарастырылған. Объектілердің динамикалық реакциясының сандық талдауы В.В. Новожиловтың қабықшаларының сызықты емес теориясы шеңберінде орындалған.*

*[2, 3] жұмыстардағы араластырылған қабық-өзекшелі құрылымның вибрациялық жағдайы және амплитудалық − жиіліктік сипаттамалары сандық есептеу жағынан зерттелді.*

*[4, 5] жұмыста С.П. Тимошенконың гипотезаларына негізделген қабық теңдеулерінің толық жүйесі келтіріледі. Айналудың инерциясын және қалыпты элементтің көлденең ауысуын ескеретін қабықшалар динамикасының нақтыланған теориясын пайдалану қазіргі заманғы техникада кеңінен қолданылатын полимерлік және композициялық материалдар ығысу деформациясына әлсіз кедергімен сипатталады, олар қабықшалардың классикалық теориясымен ескерілмейді, ал қарастырылатын тәсіл шеңберінде нөлдік емес мәндерді қабылдайды.*

***Түйін сөздер:** екі өлшемді термотұтқырсерпімді толқындар, айырымдық схеманың орнықтылығы, айырымдық есептің шешімінің жинақтылығы, индентор, деформация, тензор, кернеу.*

**УРАВНЕНИЯ НЕОСЕСИММЕТРИЧНЫХ КОЛЕБАНИЙ ОБОЛОЧЕК С ПРИСОЕДИНЕННЫМИ МАССАМИ**

М.М. Букенов, Е.М. Мухаметов, Е.А. Оспанов, С.Т. Сулейменова

*Тонкостенные оболочечные конструкции, способные выдерживать действие интенсивных внешних нагрузок, находят широкое и разнообразное применение в авиастроении, ракетной технике машиностроении, судостроении, строительстве и других отраслях народного хозяйства. Современные требования к снижению весогабаритных показателей летательных и транспортных аппаратов, промышленных и гражданских сооружений при условии обеспечения необходимой прочности к надежности сделали расчет их напряженно-деформируемого состояния одной из актуальных проблем механики деформируемого твердого тела.*

*В последнее время наметилась устойчивая тенденция к проведению комплексных теоретико-экспериментальных исследований нестационарного деформирования реальных оболочечных конструкций, состоящих, как правило, из нескольких секций и характеризующихся наличием присоединенного груза, различного рода подкрепляющих элементов, ослабляющих отверстий и других усложняющих факторов. Необходимость адекватного описания динамического поведения оболочечных систем о усложненными геометрической и реологической структурами приводит к математическим моделям, выходящим за рамки традиционных расчетных схем. Так, например, в [1] рассмотрено действие ударных волн на цилиндрической оболочке, на торцах которых находятся твердые тела с различными массами и моментами инерции. Численный анализ динамической реакции объектов выполнен в рамках нелинейной теории оболочек В.В.Новожилова.*

*Вибрационное состояние и амплитудно-частотные характеристики комбинированной оболочечно-стержневой конструкции с присоединенными массами численно исследовались в [2, 3].*

*В настоящей работе приводится полная система оболочечных уравнений, основанных на гипотезах С.П.Тимошенко [4, 5]. Использование уточненной теории динамики оболочек, учитывающей инерцию вращения и поперечный сдвиг нормального элемента, обусловлено тем, что полимерные и композиционные материалы, широко применяемые в современной технике, характеризуются слабым сопротивлением деформациям сдвига, которые не учитываются классической теорией оболочек, а в рамках рассматриваемого подхода принимают ненулевые значения.*

***Ключевые слова:** двумерные термовязкоупругие волны, устойчивость разностной схемы, сходимость решения разностной задачи, индентор, деформация, тензор, напряжения.*

МРНТИ: 50.47.29.

**Б.Б. Оразбаев[1], Д.Р. Зинағабденова[1], К.Н. Оразбаева[2], Е.А. Оспанов[3] .**

[1]Л.Н. Гумилев атындағы Еуразия Ұлттық университеті, Нұр-Сұлтан қ.

[2]Қазақ экономика,қаржы және халықаралық сауда университеті, Нұр-Сұлтан қ.

[3]Семей қаласының Шәкәрім атындағы университеті

## ГАЗДЫ ТАРАТУ ЖӘНЕ ЕСЕПКЕ АЛУ ҮРДІСТЕРІНІҢ БАСҚАРУ ЖҮЙЕЛЕРІН ТАЛДАУ, ОЛАРДЫ ЖЕТІЛДІРУ ТӘСІЛДЕРІ

***Аңдатпа:** Газды тарату және есепке алу үрдістерінің басқару жүйелерін талдау нәтижелері келтіріліп, анықсыздық пен бастапқы ақпараттың айқынсыздығы жағдайларында жұмыс жасау үшін талданған жүйелерді жетілдіру тәсілдемелері ұсынылған.*

*Газ тарату және есепке алу технологиялық үрдістерінің автоматтандырылған басқару жүйелерінің құрылымы сипатталған, мұндай жүйелердің негізгі элементтері мен ақпараттық ішкі жүйелері қарастырылған. Газды тасымалдау және тарату технологиялық үрдістерін автоматтандырылған басқару жүйелері тәжірибелік-өндірістік эксплуатациялау жайлы ақпаратты талдау нәтижесінде мұндай автоматтандыру жүйелері операторлардың көп қайталанып жасалынатын жұмыстарын минималды қылатыны анықталған, сәйкесінше*